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Effects of a Web-Based Strategic, Interactive Computer Application (*Fun Fraction*) on the Performance of Middle School Students with Learning Disabilities in Solving Word Problems with Fractions and Multiplication

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Dedication

This dissertation is dedicated to my parents for providing endless love and support, and to my husband and daughter.

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Effects of a Web-Based Strategic, Interactive Computer Application (*Fun Fraction*) on the Performance of Middle School Students with Learning Disabilities in Solving Word Problems with Fractions and Multiplication

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Abstract: The purpose of this study was to investigate the effects of a web-based strategic, interactive computer application (*Fun Fraction*) on the ability of middle school students with LD, who have mathematics goals on their IEPs, to solve word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. A multiple-probe single case research design across subjects was applied for the study. Three middle school students with learning disabilities participated in baseline, intervention, and maintenance test sessions over a 13-week period.

Findings showed that there was an experimental effect for all three students, tested on their instructional probes; students' performance improved from baseline to intervention phases after receiving instruction through *Fun Fraction*. John and Alec reached the mastery level of 80% on two of the three review days. The level of change from baseline to intervention phases ranged from 28.67% to 68.89%. Even through there was no immediacy effect for John, the trend of his data (10.33) revealed a substantial growth in general. Additionally, the percentage of data showing improvement between

baseline and intervention phases was 70% for Tiffany, 56% for John, and 100% for Alec. In particular, the improvement trend of Alec's data was statistically significant (Tau_{novlap} = 1, $p < .05$, CI 90% = .341<>1.659). All of them reached 80% accuracy percentage on their one-time maintenance tests. Regarding the three problem types of combine, partition, and compare for each representation and equation question, students struggled the most with combine representation questions and showed relatively better competence in compare equation questions. A learning-related social validity questionnaire and usability questionnaire indicated that students liked learning through *Fun Fraction* and recognized well the useful interaction design features embedded in *Fun Fraction*. Cognitive and metacognitive strategy questionnaires also indicated that students liked the represent strategy that allowed students to manipulate the rectangular area model, and students expressed positive views on the thinking process through metacognitive strategies embedded in *Fun Fraction*.

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Chapter 1: Introduction

The Individuals with Disabilities Education Act (IDEA, 2004) identifies mathematics learning disabilities (MLD) as one of several types of learning disabilities (LD). The provision of IDEA includes a disorder of the basic psychological processes and imperfect abilities in doing mathematical calculations and problem-solving. Many students with MLD demonstrate low achievement on standardized mathematics tests (e.g., below the 31st or 35th percentile), yet average or higher achievement on reading tests (Geary, Hoard, Nugent, & Byrd-Craven, 2008; Keeler & Swanson, 2001; Koontz & Berch, 1996; Lucangeli, Coi, & Bosco, 1997; Mabbott & Bisanz, 2008; Montague & Applegate, 1993; Passolunghi, 2011; Siegel & Ryan, 1989; Swanson, 1993, 1994). Prevalence studies have shown that 7% of the school-aged population has MLD (Geary, 2011; Shalev, Manor, Gross-Tsur, 2005).

Even though there is some variance on the prevalence of MLD based on different ways of identifying MLD using various measures (e.g., standardized tests or researcher-developed measures) (L. Fuchs et al., 2005), it is common that many school-aged students have insufficiently developed mathematical competences (D. Bryant, 2011; Geary, 2011). Specifically, students with LD and mathematics difficulties demonstrated significantly weak mathematical performances regarding word problem-solving, multi-step problem-solving, regrouping or renaming, and reading the value of multidigit numbers (D. Bryant, B. Bryant, & Hammill, 2000). Moreover, middle school students with LD have limitations on the concept of equivalent fractions and multiplicative relations among fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Grobecker, 2000; Mazzocco & Devlin, 2008).

Fractions as a Critical Foundation for Algebra

Successful achievement in algebra is considered to be the "gatekeeper" to postsecondary education and essential for many careers (Moses & Cobb, 2001; National Mathematics Advisory Panel [NMAP], 2008; Stinson, 2004). Mastering fractions, facilitates the learning of algebra, and thus, the learning of more advanced mathematical ideas. By 2015, nearly all states will require students to successfully complete at least one year of algebra for high school graduation (Achieve, 2004). Notably, students must learn about fractions prior to algebra instruction to be able to tackle the rigorous demands associated with this content area.

In recent years, leading professional groups have recommended important content related to fractions that students must master for algebra instruction. For example, recommendations from NMAP (2008) highlighted the importance of teaching fractions as part of a rigorous instructional program in the elementary and middle school grades. Further, NMAP indicated that knowledge of fractions should include ordering fractions, judging equivalence and relative magnitudes of fractions with unlike numerators and denominators, linking to decimals and percentages, and problem solving using representations such as a number line.

The National Council of Teachers of Mathematics (NCTM) also offered recommendations for instruction on fractions. For example, according to the Curriculum Focal Points (CFP; NCTM, 2006), students should develop an understanding of fractions and fraction equivalence in grade 3. Students should develop an understanding of and fluency with addition and subtraction of fractions in grade 5, and multiplication and division of fractions in grade 6. Resonating with the Standards (NCTM, 2000) and the CFPs (NCTM, 2006), the Common Core State Standards for Mathematics (CCSS;

Council of Chief State School Officers & National Governors' Association [CCSS & NGA], 2010) state that students should extend their previous understandings of multiplication of whole numbers to multiplying fractions in grade 5. Yet, for many middle school students with MLD, the ability to successfully compute fractions with multiplication operations remains problematic.

Difficulty with Fractions and Students with MLD

As one of the critical foundations of algebra, conceptual knowledge of fractions is considered to be an essential building block for students to successfully advance in elementary and secondary mathematics. Unfortunately, the National Assessment of Educational Progress (NAEP, 2009) found that among fourth graders, with and without disabilities, only about 25% could identify fractions closest to $\frac{1}{2}$ among four fraction values, and about 55% could identify a pictorial representation of equivalent fractions of $\frac{3}{4}$ and $\frac{6}{8}$. Among 8th graders, about 49% were able to order fractions from least to greatest (NAEP, 2007) and only about 22% of 12th graders could solve a word problem involving the division of fractions (NAEP, 2005). On an international scale, students in East Asia outperformed students in the United States in fraction understanding and knowledge (Siegler et al., 2010). Thus, performance outcomes on fractions for students, including students with disabilities, are alarming given the importance of fraction knowledge as a prerequisite for algebra instruction.

Understanding various concepts related to fractions is challenging for students with MLD (NMAP, 2008). Specifically, although dividing something in half and visually representing it with pictures or manipulatives requires relatively easy conceptual understanding (D. Bryant, Pfannenstiel, B. Bryant, Hunt, & Shin, in press), studies show that students with MLD demonstrate difficulties with rank-ordering fractions and identifying equivalent fractions (Grobecker, 2000; Mazzocco & Devlin, 2008) later grades. Students with MLD may have difficulty understanding that there are other numbers between every pair of natural numbers (Smith, Solomon, & Carey, 2005). Also, a lack of computational skills (e.g., knowledge of multiplication and division facts) presents difficulties for solving numerical expressions with fractions (Calhoon, Emerson, Flores, & Houchins, 2007). Even when solving fraction calculation problems accurately, students have been found to experience difficulty in their conceptual understanding of fraction symbols with part-whole relations (Hecht & Vagi, 2010).

This lack of conceptual understanding of and computational facility with fractions limits students' ability to solve more advanced computational problems, including ratios, rates, and proportions (Siegler et al., 2010). To remedy this situation and, thereby, better prepare students algebra, teachers must not only know about the critical concepts and skills associated with fractions but also about evidence-based interventions for teaching fractions to students with LD. Importantly, teachers who work with students with LD must have available interventions that teach procedural steps for solving fraction calculations (e.g., multiplication of fractions) to help students more successfully solve word problems, which is typically a challenge for students with MLD.

Difficulty with Word Problem Solving and Students with MLD

Mathematical problem solving represents one of the most important aspects of a school curriculum (NMAP, 2008). Unfortunately, students with MLD often struggle with word problem solving (Parmar, Cawley, & Frazita, 1996). Specifically, MLD groups demonstrated significantly lower word problem-solving scores than peers with no LD across elementary (L. Fuchs & D. Fuchs, 2002; Lucangeli et al., 1997), and secondary grades (Montague & Applegate, 1993; Montague et al., 2011). D. Bryant et al. (2000) and

D. Bryant, Smith, and B. Bryant (2008) also corroborated that students with MLD in elementary and secondary school years exhibited difficulty with word problems and multi-step problems.

Students' struggles with word problem-solving mainly stem from difficulties in reading the problem, understanding the meaning of the sentences, and understanding what is asked (Cummins, Kintsch, Reusser, & Weimer, 1988; D. Bryant et al., 2008). Additionally, word problem-solving is challenging to students with MLD due to the demanding tasks of numerous steps for the problem-solving procedure (Parmar et al., 1996). Word problem solving requires students to identify missing information, derive plans for solving for missing information, and doing calculations to find the missing information (Parmar et al., 1996; Powell, 2011). Solving word problems, which contain fractions and computation, place additional demands on students' to not only understand how to solve the problem but also to do calculations involving fractions.

More specifically, in order to solve word problems, which contain multiplying fractions, students need to have prerequisite concepts and procedures of multiplication of whole numbers and fraction equivalence (NCTM, 2010). Notably, before grade 6, students should be able to understand and be fluent with multiplication of whole numbers, understanding fraction equivalence, and understanding the area of a rectangle as the product of its length and width. In that way, in grade 6, students can learn how to visually represent multiplication with fractions and use multiplication of fractions to solve word problems (NCTM, 2010).

Theoretical Framework

The theoretical framework for the web-based strategic, interactive computer application (*Fun Fraction*) for solving word problems with fractions and multiplication is grounded in information-processing theory (Broadbent, 1958; Gagne, Yekovich, & Yekovich, 1993). This theory applies to problem-solving and multimedia learning (Anderson 1977, Spiro, & Jehng, 1990). Schema theory is also applicable to the use of schematic diagrams, hypermedia-aided learning, and computer-assisted instruction. The following sections provide an overview of information processing and schema theories to frame the intervention (*Fun Fraction*).

Information-processing theory. Based on computer processing as a metaphor, information processing theory, a cognitive processing theory (Ashcraft, 1994), describes how the human mind functions and how information is processed (Dehn, 2008) in terms of attention, working memory, and long-term memory. Related to information processing, mathematical problem-solving is complex and requires multiple cognitive processes (Mayer, 1998; Polya, 1986). For example, when teaching problem-solving, a problem solver needs to engage in the cognitive processes of encoding, inferring, applying, and responding (Mayer, 1998). However, expertise in problem execution is not sufficient for problem-solving. Many students experience metacognitive difficulties; that is, they cannot identify, monitor, and coordinate the sequenced steps necessary for problem-solving (Bransford, Sherwood, Vye, & Rieser, 1986; Gagnon & Maccini, 2007; Geary, 2004; Impecoven-Lind & Foegen, 2010; Miller & Mercer, 1997). Thus, information processing was the theoretical framework for cognitive strategy instruction in the context of word problem-solving for students with LD (Hutchinson, 1993; Ketterlin-Geller, Chard, & Fien 2008; Mayer, 1992; Montague, 1992; Montague, Enders, & Dietz, 2011; Owen & Fuchs, 2002; Uberti, Mastropieri & Scruggs, 2004).

Furthermore, the information processing theory provides the basis for multimedia learning (Mayer & Moreno, 2003). The human information processing system consists of two separate channels: an auditory/verbal channel for processing the auditory input and a visual/pictorial channel for the visual input (Mayer, 2001; Mayer & Moreno, 1998). Because the human information processing system has limited capacity (Baddeley, 1998; Chandler & Sweller, 1991; Sweller, 1999), meaningful multimedia requires careful selection of words and images, as well as how they are organized and integrated (Mayer & Moreno, 2003).

Schema theory. Schema theory links cognitive processes and constructivism (Mergel, 1998). Jean Piaget first introduced the term “schema” in 1926. Piaget defined a schema as the mental representation of an associated set of perceptions, ideas, and actions (Woolfolk, 1987). As cognitive development proceeds, new schemata are developed, and existing schemata are more efficiently organized to better adapt to a new situation (Piaget, 1926). According to schema theory of learning, abstract concepts can be best understood after a foundation of concrete information has been established (Schallert, 1982). Thus, schema theory helps to explain the use of visually framed diagrams for solving word problems (Jitendra, DiPipi & Perron-Jones, 2002; Jitendra & Hoff, 1996; Powell, 2011; Xin, Jitendra & Deatline-Buchma, 2005).

Scaffolding is essential in building upon a schema (Chalmers, 2003). Originally, scaffolding is a process through which a teacher or more capable peer helps a student in his or her zone of proximal development (ZPD; difference between what a learner can do with or without support) (Vygotsky, 1978). Especially, in the use of computers for learning (i.e., computer-assisted instruction), “interface scaffolding” (Chalmers, 2003, p. 597) is applicable; the computer can systematically fade out the scaffolding in guiding the learners’ learning process (Chalmers, 2003).

Thus, information-processing theory and schema theory have framed the web-based strategic, interactive computer application (*Fun Fraction*) for solving word problems with fractions. Namely, *Fun Fraction* involves the features of cognitive and metacognitive strategies with the use of schematic diagrams, here virtual manipulatives, embedded in computer-assisted instruction (the program enabled users to hear audio and see the video with some text animation).

Instructional Components for Teaching Students with LD

Many intervention studies have multiple and complex intervention conditions (Gersten, Chard et al., 2009). Thus, analyzing the components of each intervention allows educators to understand the essential design features of interventions; in that way, we can compare the effects of interventions that consisted of one or combinations of more than two instructional components in promoting students' mathematics achievements (Gersten et al., 2008).

Research and professional groups have emphasized several instructional components in designing effective interventions. For example, one meta-analysis study (Gersten, Chard et al., 2009) on mathematics instruction for students with LD found six effective instructional components including "explicit instruction, use of heuristics, using visual representations while solving problems, range and sequence of examples, and other instructional and curricular variables" (pp. 1210–pp. 1211) for designing curriculum and instruction. Of the six instructional components, explicit instruction, use of heuristics, use of range and sequence of examples, and use of visual representations were found to be effective for teaching fraction concepts (Shin & D. Bryant, 2012).

Explicit instruction. Explicit instruction was recommended in teaching mathematics including fractions (Gersten, Beckmann et al., 2009; Gersten, Chard et al., 2009; Jayanthi, Gersten, & Baker, 2008). Explicit and systematic instruction indicates when teachers demonstrate step-by-step strategies of how to solve problems and provide students extensive opportunities to practice (e.g., guided practice) where students could think aloud what they learned, be provided with corrective feedback on their answers, and be provided with a cumulative review (Gersten, Beckmann et al., 2009; Gersten, Chard et al., 2009; Jayanthi et al., 2008; NMAP, 2008).

Use of heuristics. Heuristic strategies may help students solve problems and organize information (Gersten, Chard et al., 2009). Cognitive strategies combined with metacognitive strategies coupled with self-regulatory strategies (e.g., self-instruction and self-questioning skills) were also recommended for solving word problems (Montague, 2008). Interestingly, Swanson (1999) found that the use of cognitive strategies was more effective when the strategies were combined with direct (explicit and systematic) instruction in teaching students with LD. The use of multiple instructional methods with a variety of strategies blended with explicit instruction was also recommended for solving mathematics problems (National Research Council; Kilpatrick et al., 2001).

Using visual representations. The use of visual representations was recommended as teachers used drawings, pictures, number lines, or graphs during their instruction and demonstrated how to solve the problems (Gersten, Beckmann et al. 2009; Gersten, Chard et al. 2009; NMAP, 2008). The U.S. Department of Education's Institute of Education Sciences (IES) practice guide on effective fractions instruction (Siegler et al., 2010) also highlighted the use of representations (e.g., number lines) including an emphasis on the development of procedural and conceptual understanding of fractions. A series of grade-level publications by NCTM (i.e., *Focus in Grade 3 through Focus in Grade 6*) endorsed the use of visual representations for teaching fractions (NCTM, 2009a, 2009b, 2009c, 2010). Specifically, research suggests that mathematical tasks include number lines, area models, and fraction-bar models (Common Core Standards Writing Team, 2011; NCTM, 2009a, 2009b, 2009c, 2010). The connection of visual representations and fractions is especially important in developing conceptual understanding of fractions and helping students engage in mathematical reasoning of how and why problem-solving works (Empson & Levi, 2011; Petit, Laird, & Marsden, 2010).

Range and sequence of examples. The use of range and sequence of examples was recommended in teaching fractions that included the features of "specified sequence/pattern of examples (concrete to abstract, easy to hard, etc.) or systematic variation in the range of examples (e.g., teaching only proper fractions vs. initially teaching proper and improper fractions)" (Gersten, Chard et al., 2009, p. 1211).

Finally, other recommendations for teaching word problems with fractions are noteworthy. For example, the concrete-representational-abstract instructional sequence was effective in teaching new fraction concepts and skills (Butler et al., 2003; Witzel, Mercer, & Miller, 2003). Namely, students with LD are recommended to use visual representations (e.g., pictures and diagrams) after learning new concepts on equivalent fractions through fraction strips and fraction bars; then, they learn how to do mathematical equations (Butler et al., 2003; Witzel, Mercer, & Miller, 2003).

NMAP (2008) also recommended, "using real-world problems to teach mathematics" (pp. 49–pp. 50). The use of real-world contexts was recommended for high school students in remedial classes for teaching fractions, basic equation solving, and function representations (NMAP, 2008).

Virtual Manipulatives for Teaching Mathematics

Several reviews and syntheses have demonstrated the effects of the use of manipulatives and virtual, visual manipulatives (Martin, 2007; Martin & Lukong, 2005) on mathematics performance when teaching mathematics to students with LD (Gersten, Chard et al., 2009; Maccini, Strickland, Gagnon, & Malmgren, 2008; Witzel, Riccomini, & Schneider, 2008). Technology has provided learners opportunities to actively engage in the manipulation of visual representations on the World Wide Web (Sayeski, 2008).

Specifically, web-based virtual manipulatives, which are interactive dynamic visual representations of concrete manipulatives, function just like mathematics manipulatives such as geoboards, base-10 blocks, and fractions bards and circles (Martin, 2007; Spicer, 2000). Virtual Manipulatives are used in introducing or reviewing mathematical concepts and skills (e.g., fractions, area and perimeter, place value, algebra) by visually representing the abstract concepts (Bouck & Flanagan, 2010; Martin & Lukong, 2005; Moyer, 2001), Contrary to static visual representations, which are pictures, sketches, or drawings, virtual manipulatives allow users to freely manipulate and slide visual representations with a computer mouse, increasing or decreasing the quantities and sizes of the visual representations (Moyer et al., 2002). Taylor (2001) stated that virtual manipulatives were beneficial to classroom learning as they relate to elementary mathematics education. Taylor said traditional classroom tools (e.g., pencils, notebooks, and texts) were still important, yet inappropriate when students needed to modify visual representations and extend their learning. Virtual manipulatives, as a means of connecting and broadening their prior knowledge, can assist students in developing mathematical notions and confronting misconceptions about challenging concepts (Taylor, 2001).

Especially, when planning lessons for students with LD who are accessing the general education mathematics curriculum, virtual manipulatives can be implemented (Maccini et al., 2008). Virtual manipulatives support students with learning difficulties who have memory problems, fail to use strategies, and have low achievement in mathematics. They can be used as a form of scaffolding: providing step-by-step directions of how to solve the problem by providing guiding questions, additional explanations for the unsure problems, immediate feedback, multiple practice opportunities, and learners' scores (Sayeski, 2008).

Recent research on mathematics also suggests that virtual manipulatives support students' learning of fractions. The interactive, web-based visual representation of dynamic objects, virtual manipulatives, represent opportunities for constructing mathematical knowledge by allowing users to engage and control the physical actions of objects (Moyer et al., 2002). For example, virtual manipulatives promoted students' learning of fractions by providing corrective verbal feedback, allowing students to flexibly change objects presented on the screen (Reimer & Moyer, 2005), creating opportunities for tracking the actions of reorganizing manipulatives (Schwartz & Martin, 2006), providing easier and faster ways to solve fractions than paper-and-pencil tools, and enhancing students' motivation to learn (Reimer & Moyer, 2002). More importantly, the multiple representations of visual materials, written words, and numerical symbols provided scaffolding for students with LD while learning fractions (Reimer & Moyer, 2002).

Statement of the Problem

Although several interventions have focused on teaching fractions to students with LD (e.g. Bottge, Heinrichs, Mehta, & Hung, 2002; Bottge, Rueda, Grant, Stephens, & LaRoque, 2010; Butler et al., 2003; Gersten & B. Kelly, 1992; B. Kelly, Carnine, Gersten, & Grossen, 1986; B. Kelly, Gersten, & Carnine, 1990; Lambert, 1996; Miller & Cooke, 1989; Test & Ellis, 2005; Woodward & Gersten, 1992), few have examined teaching middle school students with LD and no study presented grade-level fractions concepts using virtual manipulatives to teach multiplication of fractions. In the current study, the researcher developed the web-based strategic, interactive computer application (*Fun Fraction*) for teaching the concept of multiplication of fractions. CCSS (CCSS & NGA, 2010) recommended teaching multiplication of fractions in grades 5 and 6. Regarding middle school level mathematics expectations per NCTM's Focal Points (2006) and Texas Essential Knowledge and Skills (TEKS; TEA, 2012), students should be fluent with multiplying fractions by grade 6. Given the grade-level expectations and the mathematics challenges of middle school students with LD, teaching multiplication of fractions to middle school students with LD is important. Thus, implementing the web-based strategic, interactive computer application using virtual manipulatives for solving word problems with fractions and multiplication warrants research.

Purpose of the Research

The purpose of this study is to investigate the effects of the web-based strategic, interactive computer application (*Fun Fraction*) on the ability of middle school students with LD, who have mathematics goals on their IEPs, to solve word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. The following questions guided this study:

Research Questions

1. What is the effect of a web-based strategic, interactive computer application (*Fun Fraction*) on the performance of middle school students with LD, who have mathematics IEP goals, on solving word problems with fractions and

multiplication including two factors of a whole number (less than or equal to 4) and proper fractions?

2. How do middle school students with LD, who have mathematics IEP goals, maintain their mathematics performance when solving word problems with fractions and multiplication using a web-based strategic, interactive computer application (*Fun Fraction*) in 2 weeks following the intervention?
3. What is the perspective of middle school students with LD, who have mathematics IEP goals, on their ability to solve word problems with fractions and multiplication using a web-based strategic, interactive computer application (*Fun Fraction*)?
4. What is the perspective of middle school students with LD, who have mathematics IEP goals, on the use of cognitive and metacognitive strategies embedded in a web-based strategic, interactive computer application (*Fun Fraction*)?

Chapter 2: Review of Related Literature

Mathematical Problem Solving Performance of Students with Mathematics Learning Disabilities

The word problem-solving includes both arithmetic word problems and real-world problem-solving tasks (Swanson & Jerman, 2006). Research (e.g., L. Fuchs et al., 2005; Gersten, Clarke, & Jordan, 2007) has supported that word problem-solving is a predictor of MLD. Students are expected to apply grade-level number and operations and algebraic thinking within the context of real-life word problem situations (CCSS & NGA, 2010). Recently, high-stakes standardized tests like NAEP (2011) requires word problem-solving skills. Educational organizations such as NCTM (2000) and NMAP (2008) place a heavy value on problem-solving for all school-age populations. For that reason, students with MLD who are struggling with word problem-solving (L. Fuchs & D. Fuchs, 2002; Lucangeli et al., 1997; Mabbott & Bisanz, 2008; Montague & Applegate, 1993) could find these high-stakes tests to be relatively more challenging. The findings of Swanson and Jerman (2006)'s meta-analysis support that verbal problem-solving was one of the significantly deficit areas of students with MLD compared to age-matched students with no LD.

Students with MLD versus students with NLD matched on the same age or grade of students with MLD. Students with MLD demonstrated lower problem-solving performance than age or grade-matched peers with no LD (NLD-age or NLD-grade) in four studies (L. Fuchs & D. Fuchs, 2002; Lucangeli et al., 1997; Mabbott & Bisanz, 2008; Montague & Applegate, 1993). In three of the four studies, elementary students in fourth or fifth grades with MLD demonstrated significantly lower mathematics problem-solving (standardized mean difference [SMD] = -4.39, $p < .001$), more computational

(SMD = .86, $p < .001$), and more procedural errors (SMD = 1.12, $p < .001$) (Lucangeli et al., 1997) and lower scores on both complex story problems and real-world problems (SMD range = from -.42 to -.61) compared to students with NLD-grade (L. Fuchs & D. Fuchs, 2002). In Mabbott and Bisanz's study (2008), the group mean differences was smaller (SMD = -.18) than those of other studies, and even more, the two groups scored the same on tasks asking to prove why a certain product was a correct answer for a specified question. In one study for sixth to eighth graders (Montague & Applegate, 1993), students with NLD-grade significantly outperformed students with MLD on the six word problems on the Mathematical Problem Solving Assessment (Montague & Bos, 1990) (SMD = -.63, $p < .01$).

Students with MLD versus younger students with NLD matched on the mathematical ability of students with MLD. One study (Mabbott & Bisanz, 2008) compared word problem-solving performances between students with MLD and mathematical ability-matched younger students with no LD (NLD-ability). On the multiplication concept measures of word problems, students with MLD at average 11.40 years of age exhibited slightly higher performances than younger students with NLD-ability at average 9.20 years of age (SMD = 40 and .24, respectively). The word problem tests included four questions: one with irrelevant information that could be ignored; one with insufficient information to solve the problem; one requiring multiplication, addition, and comparison to solve a multistep problem; one with combining a group number into set (Mabbott & Bisanz). On another concept measure of proofs that requiring students to group manipulatives into appropriate sets to show justification of the answers to multiplication problems, students with MLD also exhibited slightly higher performances than younger students with NLD-ability (SMD = .24).

Summary. Students with MLD demonstrated significantly lower scores than students with NLD-age or NLD-grade on word problem-solving across elementary and secondary grades. The significantly lower mathematical problem-solving of students with MLD supports the definition of MLD in the IDEA (2004); IDEA regulation on the identification of specific learning disabilities states that students with specific learning disabilities should have limitations in mathematical problem-solving in a level appropriate for their age or “State-approved grade-level stands”. However, the finding of the comparison between students with MLD and younger students with NLD-ability showed that students with MLD had slightly higher problem-solving skills than younger students with NLD-ability.

Working Memory Performance of Students with Mathematics Learning Disabilities

Working memory includes components such as phonological loop, which is responsible for the storage of verbal information (e.g., word, digit forward span, and word forward span) **(**Passolunghi, 2011; Zheng, Swanson, & Marcoulides, 2011), visual-spatial, which is responsible for the storage of mental images (e.g., visual matrix, picture sequence, and mapping and directions) **(**Swanson, 1993, 1994), and central executive, which coordinates and interacts with the above two working memory components (e.g., listening sentence span, digit/sentence span, backward digit span, and operation span) (Mabbott & Bisanz, 2008; Zheng et al., 2011).

In several reviews and syntheses on the cognitive characteristics of students with MLD, researchers found working memory as the core deficit cognitive domain (Geary, 1993, 2004, 2010, 2011; Swanson & Jerman, 2006). Geary (1993) reviewed the core deficits underlying MLD. Regarding fact retrieval from memory, Geary (1993) insisted that the developmental differences of students with MLD be demonstrated in persistent deficits in retrieval, frequent errors, and unsystematic retrieval speeds. Geary (2004) continuously highlighted that students with MLD showed more errors in retrieving arithmetic facts from long-term memory than their typical peers. Swanson and Jerman's meta-analysis (2006) showed that students with MLD's significantly deficit cognitive domains included verbal working memory and visual-spatial working memory, favoring the typical peers. Students with MLD had more persistent deficits in working memory areas (i.e., phonological, visual-spatial, and central executive) compared to low-achieving students (Geary, 2010, 2011).

More importantly, Zheng et al. (2011) found that central executive processing (i.e., coordinating phonological loop and visual-spatial component; Baddeley & Hitch, 1974; Baddeley & Logie, 1999) was definitely the strongest predictor of word problem-solving over visual-spatial working memory (i.e., storage for visual-spatial images). Because central executive functioning requires inhibiting irrelevant information when integrating incoming and previously encoded information in the working memory (Bull & Espy, 2006), better executive functioning can facilitate the problem-solving process. The finding of L. Fuchs and D. Fuchs' study (2002) confirms the role of central executive functioning in solving word problems.

Students with MLD versus students with NLD matched on the same age or grade of students with MLD. In nine studies (Geary et al., 2000; Keeler & Swanson, 2001; Koontz & Berch, 1996; Mabbott & Bisanz, 2008; Passolunghi, 2011; Schuchardt et al., 2008; Siegel & Ryan, 1989; Swanson, 1993, 1994), student with NLD-age or NLD-grade outperformed students with MLD; there were significant group differences on all visual-spatial tasks and some of central executive tasks. Regarding phonological loop capacity, most studies reported no significant group differences.

Studies supported significant group differences, in particular, on central executive and visual-spatial tasks. Students with NLD-age significantly outperformed students with MLD on Digit Sentence span (SMD = -1.17, $p < .01$) and Mapping and Directions (SMD = -.64, $p < .01$) tasks as measured by the Swanson-Cognitive Processing Test (Swanson, 1995) in Keeler and Swanson's study (2001). Students with MLD had shorter working memory spans on the Backward Digit Span subtest (SMD = -1.12) of the WISC-III (Wechsler, 1991) and the operation span task (SMD = -.56) in Mabbott and Bisanz's study (2008) and on the listening span target words (SMD = -1.42, $p < .001$) and series (SMD = -.74, $p < .05$), by making significantly more intrusion errors (i.e., "non-target words within the sentences that were remembered in error" p.66) (SMD = .68, $p < .05$) than students with NLD-age in Passolunghi's study (2011).

Within the central executive tasks, students with MLD did not have significantly lower scores than students with NLD-age on sentence tasks, yet they had significantly lower scores than NLD-age on counting tasks in Siegel and Ryan's study (1989). Specifically, students with MLD and students with NLD-age had comparable scores on counting tasks at the 9-10 age level (SMD = -.30), yet students with MLD exhibited significantly lower scores on counting tasks than students with NLD-age at the 11-13 age level (SMD = -2.20).

Despite the significant group differences on visual-spatial working memory tasks, Passolunghi (2011) found that the two groups of students with MLD and students with NLD-age did not differ significantly on phonological loop tasks such as word forward span (SMD = -.14, $p > .05$) and the WISC-R (Wechsler, 1974) Digit Span (SMD = -.39, $p > .05$) tasks. Additionally, Schuchardt et al.'s study (2008) showed that the group mean differences were smaller on phonological loop (SMD range = from -.15 to -.79) and central executive tasks (SMD range = from -.30 to -.60) than on visual-spatial tasks (SMD range = from -.28 to -1.20).

Regarding the phonological loop and central executive working memory, some studies reported contrasting findings compared to the above studies. Geary et al. (2000) also found that there were no significant group differences on the phonological loop task such as the forward digit span (SMD = -.44), yet they found no significant group differences on the central executive task such as the backward digit span tasks (SMD = -.58). Koontz and Berch (1996) showed that students with MLD demonstrated limits in their phonological loop capacity with significantly smaller digit spans (SMD = -1.00, $p < .05$) and letter spans (SMD = -1.07, $p < .01$) than students with NLD-grade. In Swanson's studies (1993, 1994), where the two groups were compared on verbal and visual-spatial tasks, students with MLD performed significantly lower than students with NLD-age on both tasks (*p*s < .05).

To summarize, students with MLD performed less well than students with NLD-age or NLD-grade on most measures of working memory, in particular, visual-spatial tasks. Most studies supported that the two groups did not have significant different phonological loop capacity, and within the central executive tasks, students with MLD showed significant limitations on the counting task compared to students with NLD-age or NLD-grade.

Students with MLD versus younger students with NLD matched on the mathematical ability of students with MLD. In three studies (Keeler & Swanson, 2001; Mabbott & Bisanz, 2008; Swanson, 1993) that examined differences in working memory characteristics between students with MLD and younger students with NLD-ability, the two groups showed no significant group differences on central executive and visual-spatial tasks. For example, students with MLD and younger students with NLD-ability performed similarly (*p*s > .01), while students with MLD had slightly longer spans on the Digit Sentence span task (SMD = .16) and scored lower on the Mapping and Directions task (SMD = -.05) in Keeler and Swanson's study (2001). Additionally, in Mabbott and Bisanz's study (2008), the two group means' effect sizes ranged from small to medium, while students with MLD had slightly longer spans on operation span tasks (SMD = .06) and shorter spans on the WISC-III (Wechsler, 1991) Backward Digit Span subtest (SMD = -.46).

Regarding verbal and visual-spatial tasks, while students with MLD performed better than younger students with NLD-ability on most of the retrospective and prospective verbal and visual-spatial tasks, students with MLD had significantly higher scores than younger students with NLD-ability ($p < .05$) on retrospective verbal (i.e., rhyming, story recall, and semantic association) and retrospective visual-spatial tasks (i.e., visual matrix) (SMD range = from .03 to .65) except for the picture sequence task in Swanson's study (1993). There were no significant group differences ($p > .05$), however, on prospective verbal (i.e., auditory digit sequence, phrase sequence, and semantic categorization) and prospective visual-spatial tasks (i.e., mapping/directions, spatial organization, and nonverbal sequencing) (SMD range = from .01 to .56).

Summary. To summarize, regarding working memory differences between the two groups, students with MLD showed significantly lower visual-spatial and central executive working memory than students with NLD-age or NLD-grade. Within the findings on central executive working memory, however, Siegel and Ryan's study (1989) reported that students with MLD exhibited no significant differences on the non-numerical verbal working memory tasks (e.g., sentences tasks), yet significant deficits on the numerical verbal working memory tasks (e.g., counting tasks). This finding supports the deficit of domain-specific knowledge that is related to numbers among students with MLD (Raghubar, Barnes, & Hecht, 2010). Students with MLD were likely not to have a deficit on a language-related working memory task (e.g., phonological loop tasks) that measures processes similar to reading; yet they had limitations on a working memory task that includes counting and remembering the products of those counts (Siegel & Ryan, 1989). Contrary to the comparison between students with MLD versus students with NLD-age or NLD-grade, there were no significant group differences between students with MLD and younger students with NLD-ability on central executive and visual-spatial working memory among elementary school populations.

Metacognitive Performance of Students with Mathematics Learning Disabilities

Metacognition refers to “knowing about knowing” and knowledge and awareness of one’s cognitive processes (Wong, 1999). If students are aware of their “knowing” processes, they can retain information easily and proceed (Paris, Lipson, & Wixson, 1983). Several studies (Desoete & Roeyers, 2002; Garrett, Mazzocco, & Baker, 2006; Lucangeli et al., 1997; Montague & Applegate, 1993) showed that many students with LD lack metacognitive skills such as self-monitoring and self-evaluating one’s performances (Desoete & Roeyers, 2002; Bannert & Mengelkamp, 2008). NCTM (2000) and CCSS (CCSS & NGA, 2010) advocate learning mathematical problem-solving skills that need higher-order thinking strategies such as metacognition (Mayer, 1998, Sweeney, 2010). That is, to solve word problems, students should have “knowledge of when to use, how to coordinate, and how to monitor various skills” (p. 53) as well as the cognitive skills of instructional objectives and heuristic strategies (Mayer, 1998).

Students with MLD versus students with NLD matched on the same age or grade of students with MLD. Students with MLD scored significantly lower than students with NLD-grade on measures of metacognition and used a limited number of metacognitive strategies in four (Desoete & Roeyers, 2002; Garrett et al., 2006; Lucangeli et al., 1997; Montague & Applegate, 1993) studies. In the study by Desoete and Roeyers (2002), students with MLD had significantly lower scores than students with NLD-grade on the EPA 2000 (De Clercq et al., 2000) (SMD = -2.72, $p < .05$). Students with MLD predicted and evaluated significantly less well than students with NLD-grade on all prediction and evaluation tasks of EPA 2000 (De Clercq et al., 2000) (SMD = -1.85 and -1.88, *p*s < .05, respectively). Regarding the task difficulties, students with MLD had significantly lower scores on both prediction and evaluation tasks designed for grades 1, 2, and 3 (SMD range = from -1.69 to -2.05, *p*s < .05) than did students with NLD-grade. On tasks designed for grade 4, however, students with MLD had significantly better scores on prediction (SMD = .41, $p < .05$) and slightly better scores on evaluation (SMD = .01, $p > .05$) than students with NLD-grade.

In the longitudinal study by Garrett et al. (2006), during second and third grade, students with MLD evaluated their performance significantly less accurately (SMD = -1.34 and -1.76) and had fewer "sure" correct (SMD = -1.77 and -2.43) and more "not sure" and "sure" incorrect responses (SMD range = from .85 to 2.14) than students with NLD-grade (*p*s < .001), yet the two groups did not differ on total "sure" (SMD = -.54 and -1.09) and "not sure" correct responses (SMD = -.45 and -.12). Moreover, regarding prediction accuracy, students with MLD predicted that they could correctly solve significantly fewer calculation problems and less accurately predicted the number of items that they could solve correctly (SMD range = from -.90 to -2.29, *p*s < .001) than students with NLD-grade during third and fourth grade.

In the study by Lucangeli et al. (1997), students with MLD with poor problem-solving skills had significantly lower levels of metacognitive awareness than students with NLD-grade (SMD = -.73, $p < .001$). Students with MLD were more likely to believe that the size of numbers in a problem was an indicator of the difficulty of the problem (SMD = 1.10, $p < .001$). In Montague and Applegate's study (1993), students with MLD reported significantly fewer problem representation strategies (SMD = -1.40, $p < .01$) than students with NLD-grade, yet the two groups demonstrated no significant differences for the number of strategies regarding strategy knowledge, strategy use, and strategy control (SMD range = from -.39 to -.73) as well as for the quality of strategies with IQ as the covariate (SMD range = from -.24 to -.74) on the Mathematical Problem Solving Assessment (Montague & Bos, 1990).

In brief, students with MLD had significantly lower metacognitive prediction and evaluation capacities than students with NLD-grade. Students with MLD less accurately predicted and evaluated their problem-solving performances compared to students with NLD-grade and used fewer strategies and lower quality strategies on the problem-solving test.

Students with MLD versus younger students with NLD matched on the mathematical ability of students with MLD. In the study by Desoete and Roeyers (2002), no group differences were found between students with MLD and younger students with NLD-ability on any prediction (SMD = -.28) and evaluation (SMD = -.11) skills in the EPA 2000 (De Clercq et al., 2000). Moreover, there were no significant group differences on prediction and evaluation tasks for grades 2, 3, and 4 (SMD range =

from -.02 to -.37, *p*s > .05), while students with MLD had significantly lower prediction and evaluation scores for grade 1 (SMD = -.82 and -.60, *p*s < .05, respectively) than younger students with NLD-ability. Regarding different mathematical problem-solving tasks, students with MLD showed significantly lower prediction performance on the number knowledge, mental arithmetic, and procedural calculation (SMD range = from -.37 to -.50, *p*s < .05) and significantly lower evaluation performance on number system knowledge and procedural calculation (SMD = -.42 and -.38, *p*s < .05, respectively) compared with younger students with NLD-ability; no significant group differences were found on other prediction (i.e., numeral and operation symbol comprehension and word problem) and evaluation tasks (i.e., numeral and operation symbol comprehension, mental arithmetic, and word problem) (*p*s >.05).

Summary. In summary, students with MLD had significantly lower metacognitive abilities than students with NLD-grade on prediction and evaluation tasks. Specifically, students with MLD predicted and evaluated their problem-solving performances significantly less accurately than their peers with NLD-grade. Students with MLD also had limited problem-solving strategies compared to students with NLD-grade. Additionally, comparing students with MLD and younger students with NLD-ability, the two groups did not have significant group differences on all the metacognitive tasks.

Designing Fraction Instruction: NCTM Standards, Instructional Components and Effects of Fraction Interventions for Students with Learning Disabilities

NCTM standards. NCTM standards (2000) include Content Standards (i.e., Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) and Process Standards (i.e., Problem Solving, Reasoning and Proof, Communication, Connections, and Representation) and they guide what teachers have to teach. Of 13 (Bottge, 1999; Bottge et al., 2002, 2010; Butler et al., 2003; Courey, 2006; Gersten & B. Kelly, 1992; Joseph & Hunter, 2001; B. Kelly et al., 1986, 1990; Lambert, 1996; Miller & Cooke, 1989; Test & Ellis, 2005; Woodward & Gersten, 1992) studies that focused on teaching fractions to students with LD, the most frequently used NCTM Content Standard was Number and Operations. Most of the 13 studies (n = 11) focused on fraction computation. Of these 11 studies, eight (Bottge; Bottge et al., 2002, 2010; Gersten & B. Kelly, 1992; Joseph & Hunter, 2001; B. Kelly et al., 1986, 1990; Test & Ellis, 2005) included adding and subtracting fractions; one (B. Kelly et al., 1986) included multiplication of fractions as well as the addition and subtraction of fractions. Five studies included equivalent fractions (Bottge et al., 2010; Gersten & B. Kelly, 1992; B. Kelly et al. 1986; Woodward & Gersten, 1992) beyond fractions computations. Three studies (Bottge; Bottge et al., 2002, 2010) also included the Measurement standard of converting measurement equivalents of inches to feet.

The studies focused on several Process Standards, most commonly Representations. Nine studies (Bottge et al., 2010; Butler et al., 2003; Courey, 2006; Gersten & B. Kelly, 1992; B. Kelly et al., 1986, 1990; Lambert, 1996; Miller & Cooke, 1989; Woodward & Gersten, 1992) applied Representations by constructing tables and using diagrams, showing spreadsheet displays, or using numerical symbols (e.g., __ + __ = [ ]) to translate a fraction picture to a numerical equation. The next commonly used Process Standard was Problem Solving. Here six (Bottge, 1999; Bottge et al., 2002, 2010; Butler et al., 2003; Courey, 2006; Lambert, 1996) used Problem Solving by focusing on unknown solution methods via contextualizing problems or applying visual representations, four (Bottge, 1999; Bottge et al., 2002, 2010; Butler et al., 2003) focused on Connections by systematically engaging learners' prerequisite skills and concepts and linking contextualized experiences of buying a product or using white beans to facilitate rational number counting, and four (Bottge, 1999; Bottge et al., 2002, 2010; Joseph & Hunter, 2001; Lambert, 1996; Test & Ellis, 2005) included Communication by having students reflect mathematical thinking aloud through discourse (discussions and explanations) with peers or teachers.

Instructional components and effects of fraction interventions. The instructional components including visual representations, explicit and systematic instruction, range and sequence of examples, heuristic strategies, and use of real-world problems were analyzed in the total of fourteen studies. Additionally, the effects of fraction interventions consisting of these instructional components were also described.

Visual representations. The use of visual representations indicated when teachers used drawings, pictures, number lines, or graphs during their instruction by demonstrating how to solve the problems (Gersten, Beckmann et al., 2009; Gersten, Chard et al., 2009; NMAP, 2008). The most commonly used instructional component was visual representations. Three studies (Courey, 2006; Miller & Cooke, 1989; Woodward & Gersten, 1992) used visual representations exclusively. Visual representations were used as the only "teachers" in two studies (Miller & Cooke, 1989; Woodward & Gersten, 1992) of the three studies. While implementing videodisc instruction that included special graphics within the program, teachers and researchers taught students how to analyze and represent fractions (e.g., fraction pie models) shown in the video (Miller & Cooke, 1989; Woodward & Gersten, 1992). The two single-group pre-/posttest studies showed improvement on the posttest of fraction computation with a highly large effect of 6.91 (Miller & Cooke, 1989) and 3 (Woodward & Gersten, 1992).

In the study by Courey (2006), researchers taught “half” word problems by implementing procedural instruction through visual representations (e.g., circles) and providing additional conceptual supplements using conceptually laden language. Although both procedural and conceptual supplements were effective in improving students’ problem solving with fractions (effect size range = from -.18 to 2.15), the group receiving the combined treatment of procedural and conceptual supplement did not outperform the group receiving only procedural group (effect size range = from -1.4 to .89).

Visual representations and range and sequence of examples. In one study (Butler et al., 2003), while applying concrete-representation-abstract instruction, teachers used concrete and visual representations in conjunction with sequence of examples. Range and sequence of examples indicated the features of “specified sequence/pattern of examples (concrete to abstract, easy to hard, etc.) or systematic variation in the range of examples (e.g., teaching only proper fractions vs. initially teaching proper and improper fractions)” (Gersten, Chard et al., 2009, p. 1211). Specifically, introducing fraction concepts and skills to students, teachers used concrete objects such as fractions strips and folded construction paper for understanding of equivalent fractions, beans to represent the denominator of equivalent fractions, and other commercial fraction circles and representational drawings in a sequential way. Students also learned how to use those manipulatives and drawings while solving fraction problems.

In Butler et al. (2003), the treatment group, which primarily comprised students with learning disabilities, receiving concrete-representation-abstract instruction in general education mathematics classes, after 10 lessons (45 minutes per session) performed better than the group with only representation-abstract instruction on all measures (effect size range = from -.55 to .88), and performed significantly better (effect size = 1.05, p < .0005) on the Quantity Fractions subtest, demonstrating a significant increase in the conceptual understanding of fraction equivalency. The effects of concrete-representation-abstract instruction on other subtests of *Brigance*, including Area Fractions and Abstract Fractions, and on the researcher-developed measures of Word Problems and Improper Fractions ranged between small and moderate (effect size = .13 to .33).

Visual representations, explicit and systematic instruction, and range and sequence of examples. Three studies (Gersten & B. Kelly, 1992; B. Kelly et al., 1986, 1990) implementing visual representations did so with two other instructional components: explicit and systematic instruction and range and sequence of examples. Especially, explicit and systematic instruction indicated that teachers demonstrated step-by-step strategies of how to solve problems, provided students extensive opportunities to practice (e.g., guided practice) where students could think aloud what they learned, provided corrective feedback on students' answers, and provided cumulative review (Gersten, Beckmann et al., 2009; Gersten, Chard et al., 2009; Jayanthi et al., 2008; NMAP, 2008).

In three group studies (Gersten & B. Kelly, 1992; B. Kelly et al., 1986, 1990), implementing the videodisc instruction (Systems Impact, Inc., 1986) for secondary students, interventionists presented fraction pictures embedded in the videodisc software to teach basic fraction concepts and skills. More importantly, teachers implemented systematic instruction and taught adding fractions with a step-by-step strategy. For example, students first translated fraction pictures into numerical equations. Teachers introduced the procedural rule of adding fractions after students gained conceptual understanding of adding fractions through fraction pictures. Moreover, lessons addressed the differences between addition and multiplication, and after students had had enough practices to be able to discriminate these two problems, their skills were integrated with other types of fraction problems; fraction terms such as numerator and denominator were presented in a separate way in order to prevent students' confusion over the terminology (B. Kelly et al., 1986, 1990).

In the three studies (Gersten & B. Kelly, 1992; B. Kelly et al., 1986, 1990), teachers also presented a wide range of examples by teaching how to read and write both proper and improper fractions from the beginning to avoid misconceptions of fractions and by providing fraction problems with the unknown value on both the left and right sides of the equal sign (B. Kelly et al., 1990). The feature of range of examples was distinct from instruction for the control group. In the control group (e.g., basal instruction), teachers only presented proper fractions (e.g., fractions less than 1), and the unknown value was always presented on the right side of the equal sign (B. Kelly et al., 1990). Thus, secondary students struggling with mathematics receiving videodisc instruction via visual representations, explicit and systematic instruction, and range and sequence of example outperformed students receiving the basal (textbook) curriculum on curriculum-referenced tests of fraction computation skills with a large effect size of 1.04 for B. Kelly et al. (1986) and a fairly large effect size of .62 ($p < .01$) for B. Kelly et al. (1990).

Heuristic strategies. Heuristic strategies indicated cognitive strategies that anchors students to solve problems and organize information (Gersten, Chard et al., 2009). Cognitive and metacognitive process of learning with self-regulatory strategies includes self-questioning skills for solving problems (Montague, 2008). Two single-subject design studies (Joseph & Hunter, 2001; Test & Ellis, 2005) used heuristic strategies. Joseph and Hunter applied cue cards for improving the number of fraction problems calculated correctly for three eighth-grade students with learning disabilities. The PND of the three students were high: 100% (Rob), 94% (Mark), and 93% (Nick). However, the three students had diverse cognitive abilities and responded differently; the high-average planner (Rob) and average planner (Nick) showed the most stable performance, with a mean increase of 71% and 54%, respectively, from baseline to intervention. The below-average planner (Mark) fluctuated in performance, with a mean increase of 44% from baseline to intervention. Overall, even after removing cue cards during the maintenance phase, these students maintained their performance on the fraction test, when implemented once a week over a 3-week period.

In another study, Test and Ellis (2005) worked with 6 eighth grade students with disabilities using the LAP self-regulatory mnemonic strategy for fraction computation problems. LAP stands for "**L**ook at the sign and denominator. **A**sk yourself the question: Will the smallest denominator divide into the largest denominator an even number of times? **P**ick your fraction type." (Test & Ellis, 2005, p 14). All six students' PNDs on both the LAP Fraction strategy and LAP Fraction intervention test were 100%. Results indicated a functional relationship between implementing LAP Fractions and students' acquisition of both the LAP Fraction strategy and their application of the strategy to adding and subtracting fractions. Five of six students mastered both skills and maintained their performance over 6 weeks.

Heuristic strategies and visual representations. One study (Lambert, 1996) used heuristic strategies, which were implemented with visual representations. Teachers taught eight steps of a cognitive strategy to 9^{th}–12^{th} students with LD by using a cue card and visualizing problems. The eight steps included: Read (for understanding); Paraphrase (your own words); Visualize (a picture or a diagram); State the problem; Hypothesize; Estimate; Calculate; and Self-check. Even though there was a fairly large effect size of .62, no significant difference was found between the cognitive strategy group and the textbook instruction group on a problem-solving fractions test.

Use of real-world problems. Use of real-world problems indicated when teachers apply contextualized problems to teach mathematical problem-solving (Gersten, Beckmann et al., 2009; Gersten, Chard et al., 2009; NMAP, 2008). Two studies (Bottge, 1999; Bottge et al., 2002) used video-based real-world problems, so-called video-based anchored instruction or enhanced anchored instruction. In Bottge (1999), low-achieving secondary students and students with LD received video-based anchored instruction via contextualized problems, *The 8th Caller* and *Bart's Pet Project*, and were encouraged to share their problem-solving solutions on the challenge problems on the video. The anchored instruction allowed students to solve real-world problems in authentic contexts that are embedded in the short videos. Students were also encouraged to discuss and collaborate together to solve subproblems within the videos. The anchored instruction group outperformed the word problem instruction group on a contextualized test, including adding and subtracting with fractions (effect size = 6.43); yet there was only a small or no effect size on the word problem (effect size = .19) and computation tests (effect size = -.27).

In another study (Bottge et al., 2002), secondary students with LD received enhanced anchored instruction. The instruction consisted of 8-minute contextualized video problems, *Fraction of the Cost*, and a hands-on problem of planning and building wooden benches (Bottge et al., 2002). Instruction took place either in the general mathematics classrooms and technology education classroom. Findings showed that students receiving enhanced anchored instruction outperformed students receiving word problem instruction on the contextualized problem test (effect size = 1.72). Three of four students with disabilities receiving enhanced anchored instruction showed modest improvement on the fraction word problem test. Yet, in general, there were no intervention effects on computation (effect size = -1.22) or word problem (effect size = .23) tests.

Use of real-world problems and visual representations. One study (Bottge et al., 2010) included real-world problems combined with visual representations. This study implemented videodisc software that included pictorial representations in combination of video-based enhanced anchored instruction. In Bottge et al., secondary students with LD received enhanced anchored instruction. The instruction consisted of 8-minute contextualized video problems, *Fraction of the Cost,* and a hands-on problem of planning and building a rollover cage. Instruction took place either in the general mathematics classrooms and the resource room. In Bottge et al. (2010), a combination of videodisc instruction and enhanced anchored instruction had a moderate effect on students' performance on the computation test (effect size = .40). Yet, there were no group differences between the combination instruction group and the enhanced anchored instruction group on the word problem test (effect size = -.13).

Summary. To sum up, of the standards established by the NCTN in 2000, Number and Operations was the most frequently used Content Standard, and Representation was the most frequently used Process Standard in the 13 studies that focused on teaching fractions to students with LD. Studies indicated that fraction interventions consisting of several instructional components (e.g., concrete and visual representations, range and sequence of examples, explicit and systematic instruction, heuristic strategies, and use of real-world problems) led to improvements in the fraction concepts and skills of students with LD.

Strategy Instruction in Word Problem-Solving for Middle School Students with Learning Disabilities

Strategy instruction indicates teaching specific strategies of cognitive and metacognitive processes by focusing on the rules and processes for solving word problems (D. Bryant, B. Bryant, Williams, Kim, & Shin, 2012; Swanson & Beebe-Frankenberger, 2004). The common effective strategy elements include the use of "a memory device such as first-letter mnemonic strategies, the familiar action verb (e.g., "*Read* the problem") to prompt students to use the strategy, and sequenced steps to help students remember and recall the process" (Maccini et al., 2008, p. 7). Several reviews and meta-analysis (Maccini et al., 2007, 2008; Maccini, McNaughton, & Ruhl, 1999; Montague, 1997a; Montague & Dietz, 2009; Swanson, 1999, Swanson & Sachse-Lee, 2000) found that strategy instruction is effective in teaching problem-solving for students with LD. Especially, when teaching word problem-solving for middle school students with LD, strategy instruction involving cognitive strategy instruction (e.g., Case, Harris, & Graham, 1992; Joseph & Hunter, 2001; Maccini & Ruhl, 2000; Montague, 1992, 2007, 2008; Montague et al., 2011; Montague, Applegate, & Marquard, 1993; Naglieri & Johnson, 2000) and schema-based instruction (e.g., Hutchinson, 1993; Jitendra et al., 1999, 2002; Na, 2009; Xin et al., 2005) have been found to be effective.

Cognitive strategy instruction. Cognitive strategy instruction and direct instruction share the instructional approach grounded by behavioral theory, yet cognitive strategy instruction is also based on cognitive theory (Montague & Dietz, 2009). That is, cognitive strategy instruction is based on information processing theory, emphasizing how individuals process their encoded information (D. Bryant et al., 2012). Students with LD often have limited strategies, in particular, on tasks requiring higher level processing (Montague, 2008). To help students with LD to solve word problems confidently, teachers can teach a cognitive routine using explicit and structured instruction as if students are good problem solvers (Montague, 2008; Montague et al., 2011). The goal of cognitive strategy instruction is to teach students cognitive and metacognitive strategies to enhance their problem-solving process (Montague, 2008; Montague & Dietz, 2009).

Several researchers (Case et al., 1992; Joseph & Hunter, 2001; Maccini & Ruhl, 2000; Montague, 1992; Montague et al., 1993, 2011; Naglieri & Johnson, 2000) investigated the effects of cognitive strategy instruction in teaching word problems for middle school students with LD. For example, Montague (1992) investigated the effects of a 7-setp cognitive strategy instruction embedded with metacognitive strategy (SAY, ASK, CHECK) in solving 1-, 2-, and 3-step word problems for six students with LD in grades 6 to 8. The cognitive strategy routines were (a) read the problem, (b) paraphrase the problem, (c) draw the problem, (d) create a plan to solve the problem, (e) predict/estimate the answer, (f) compute the answer, and (g) check the answer. In the single-case multiple-baseline study, the word-problem solving of middle school students with LD improved from the baseline to the intervention phase, and the results showed a combination of both cognitive and metacognitive strategies were more effective than using their cognitive strategy alone.

Montague et al. (1993) also investigated the effects of the same 7-step cognitive-strategy, SAY-ASK-CHECK metacognitive strategy, and combination of cognitive and metacognitive strategy instruction for 72 students with LD on 1-, 2-, and 3-step word problem-solving. The results of the quasi-experimental group design showed that students who received cognitive and combined cognitive-metacognitive conditions improved more than students who received metacognitive strategy instruction alone on the problem-solving posttest.

Case et al. (1992) implemented a multiple-baseline across subject single-case research where four fifth and sixth graders with LD received cognitive strategy instruction for solving one-step addition and subtraction word problems. The problem-solving strategies included five steps: "(a) read the problem out loud?; (b) look for important words and circle them; (c) draw a picture to help tell what is happening; (d) write down the math sentence; (e) write down the answer" (p. 6). Additionally, students were taught how to use self-instruction strategies including self-questioning about planning and a strategy to use for word problem-solving. Most students improved on both addition and subtraction problems compared with their performance during the baseline phase to that of the intervention phase.

In Naglieri and Johnson (2000)'s single-case study, 19 students, most of who were identified with LD in grades 6 to 8 received cognitive strategy instruction that encouraged students to learn a planning strategy for arithmetic computation through self-reflection and students' verbalization. The results showed that students with poor planning had the greatest gains (effect size = 1.4) from baseline to intervention phase.

Maccini and Ruhl (2000)'s multiple-baseline across subjects study focused on teaching graduated instructional sequences combined with a mnemonic strategy. Three eighth grade students with LD who learned a mnemonic problem-solving strategy, STAR (i.e., **S**earch the word problem, **T**ranslate the problem, **A**nswer the problem, **R**eview the solution) with a graduated instruction sequence (i.e., concrete, representational, and abstract) improved on their probes. Specifically, the mean percentage of correct problem representation and solution increased from baseline to intervention phase after STAR strategy with a graduated instruction sequence in subtracting integers with word problem-solving.

Joseph and Hunter (2001) conducted a multiple-baseline across subjects single-case study for three eighth grade students with LD, implementing cue cards in solving basic addition and subtraction fraction problems with common and uncommon denominators and with three problem-solving instances accompanied by numerical representations. Contrary to the finding by Naglieri and Johnson (2000), the above-average cognitive planner showed the highest improvement from baseline to intervention phase (i.e., 71 percent) and the below-average cognitive planner showed the lowest improvement from baseline to intervention phase (i.e., 44 percent). Researchers concluded that some students with LD needed more specific strategies for planning for problem-solving in addition to the use of cue cards.

More recently, Montague et al. (2011) conducted a randomized controlled trial study in inclusive general education math classes. The results showed that 32 middle school students with LD who received the cognitive strategy instruction *Solve It!* significantly improved in problem-solving performance compared to those of the comparison group of 46 middle school students with LD who received typical mathematical instruction. *Solve It!* incorporated cognitive and metacognitive processes (self-regulation through self-instruction, self-questioning, and self-monitoring); the cognitive process routines included read, paraphrase, visualize, hypothesize, estimate, compute, and check, and the corresponding self-regulation strategies were SAY (self-instruction), ASK (self-questioning), and CHECK (self-monitoring).

Summary. To sum up, studies to improve word problem-solving for middle school students with LD showed the effects of the use of cognitive strategy instruction. Two studies (Joseph & Hunter, 2001; Maccini & Ruhl, 2000) implemented cognitive strategy instruction in the form of a mnemonic strategy. When applying cognitive strategy instruction, most studies embedded metacognitive strategies within the cognitive process routines (Case et al., 1992; Montague, 1992; Montague et al., 1993, 2011; Naglieri & Johnson, 2000). Moreover, studies (Montague, 2011; Montague et al., 1993) reported that middle school students with LD improved more on their problem-solving tests when they received the combined instruction of both cognitive and metacognitive strategies.

Schema-based instruction. In the view of sociocultural perspectives, schema occurs due to social interactions between individuals and their environment (McVee, Dunsmore, & Gavelek, 2005). Contrary to cognitive scientists' view of a dualistic approach for schema, emphasizing cognitive structures within individuals apart from the world (McVee et al., 2005), meaning exists in embodied forms, such as words and images, within our relationships, across experiences, conversations, and people (Gee, 2004). By way of schematic learning, students can utilize a "third space", such as visual representations, as means of knowledge-sharing with teachers or their peers (Gutiérrez, Baquendano-Lopez, Turner, 1997), thereby embodying social and cultural constructs (McVee et al., 2005). Based on schematic learning and teaching, schema-based instruction was intended to help students establish and expand their conceptual knowledge where schemata are focused (Jitendra et al., 2009). To facilitate students' domain-specific conceptual and procedural understanding (Hutchinson, 1993), problem-solving instruction such as schema-based instruction was suggested for middle school students with LD (Hutchinson, 1993; Jitendra et al., 1999, 2002; Na, 2009; Xin et al., 2005). Problem schemata acquisition enables learners to use a representation to solve problems that have different features but similar structures (Sweller, Chandler, Tierney, & Cooper, 1990). Compared to general strategy instruction, schema-based instruction focused on systematically teaching students to identify different types of structures, use schematic diagrams to represent problems, and link the diagrams to math sentences in order to solve problems (Powell, 2011; Xin et al., 2005). Additionally, compared to cognitive strategy instruction, which also uses diagrams, schema-based instruction emphasizes the semantic relations in a problem to translate and solve the problem (Jitendra et al., 2002).

Five studies (Hutchinson, 1993; Jitendra et al., 1999, 2002, Na, 2009; Xin et al., 2005) targeted teaching word problem-solving to middle school students with LD and investigated the effects of schema-based instruction. For example, Hutchinson (1993) conducted both a single-case, multiple-baseline study and a group design study by teaching 20 students with LD in grades 8 to 10 to solve algebra problems. In the intervention condition, the schema-based problem representation for three different mathematical structures (i.e., relational, proportion, and two-variable and two-equation) was embedded within the application of a 10-step cognitive strategy routine with metacognitive strategies. The results showed that students' improved considerably in algebra problem-solving improved and maintained this ability at 6 weeks following the intervention. Additionally, the group design study results showed that students receiving the cognitive strategy instruction using schematic diagrams improved significantly on their posttest compared to the comparison group.

Jitendra et al. (1999) investigated the effects of schema-based strategy on one- and two-step addition and subtraction word problems for four sixth and seventh grade students with LD. Results showed that all four students improved from baseline to intervention phases on one- and two-step word problems. All four students' performances were maintained on two-step word problems (Mean = 86% correct) at two and four weeks following the intervention. Generalization of the strategy was also earned on one-step word problems.

Jitendra et al. (2002) conducted a multiple-probe single-case study on schema-based instruction for four eighth graders with LD in solving multiplication and division word problems. The schema-based instruction lasted for 35–40 minutes per session for a total of 18 sessions, presenting two different conditions: problem schemata identification and problem solution. In enhancing the conceptual understanding of how to represent each problem as a diagram, students received training on schemata identification with story situations. Then, in developing procedural knowledge, students learned how to set up the problem with an unknown quantity and apply equivalent fraction rules. Results showed that all four students reached mastery level (i.e. 100% in two sessions) and their problem-solving performance was maintained up to 10 weeks following the termination of the intervention. The effect was also generalized to novel and more complex multi-step word problems for all four students.

Xin et al. (2005) conducted a randomized controlled trial study for 18 students with LD in grades 6 to 8 in learning how to solve ratio and proportion word problems involving the multiplication and division of fractions. The intervention occurred three to four times a week for 12 sessions; each session lasted about 60 minutes. Both group of students learned the general problem-solving skills of reading to understand, representing the problem, planning, solving, and checking the answer. However, students receiving schema-based instruction learned how to identify the problem type and use a schema diagram to represent and solve the problem, while students receiving general strategy instruction learned how to draw semi-concrete pictures to present the problem and solve the problem. Results showed that students receiving schema-based instruction improved significantly as compared to students receiving general strategy instruction on posttest, maintenance, and generalization tests (*p*s < .01).

More recently, Na (2009) investigated the effects of schema-based instruction for four sixth and seventh grade students with LD on word problem-solving over 13 weeks with 12 session intervention days. Students learned two conditions of problem schemata and problem solution instruction with two different word problem types (i.e., multiplicative compare and vary problems). All four students improved from baseline to intervention, surpassing the mastery level of 70% on six consecutive tests (mean = 86%). Students' performances were generalized to the real word problems. The intervention effects were also maintained on both the dependent measure (mean = 88.8%) and generalization measure (mean = 96.3%).

Summary. In summary, several studies (Hutchinson, 1993; Jitendra et al., 1999, 2002, Na, 2009; Xin et al., 2005) showed that schema-based instruction was effective in teaching word problem-solving for middle school students with LD. The use of visual representation in teaching the concepts and skills of fractions for students with LD is highly effective and recommended (Butler et al., 2003; Courey, 2006; Gersten, Beckmann et al., 2009; Gersten, Chard et al., 2009; Gersten & B. Kelly, 1992; B. Kelly et al., 1986, 1990; Miller & Cooke, 1989; Woodward & Gersten, 1992). Especially, when teaching word problem-solving, the selection of a visual representation appropriate for the problem structure is also important (Woodward et al., 2012). Thus, schema-based instruction that emphasizes how to identify the mathematical problem structures, namely problem types, and use specific diagrams to represent the problems (Xin et al., 2005) addresses the key for solving word problems.

Using Virtual Manipulatives for Specific Mathematics Applications

Two types of representations exist on the World Wide Web: static visual representation and dynamic visual representation (Spicer, 2000). Static visual representations are pictures like visual images in books, drawings on an overhead projector, and sketches on a chalkboard; dynamic visual representations of concrete manipulatives are visual images on computers that are just like pictures in books, drawings on an overhead projector, and sketches on a chalkboard but can be manipulated in the same ways that a concrete manipulative can (Moyer, Bolyard, & Spikell, 2002). Just as students can slide, flip, and turn a concrete manipulative by hand, by using a computer mouse they can actually do the same to the dynamic representation as if it were a three-dimensional object (Moyer et al., 2002). Virtual manipulatives refer to any computer-generated image that appears on a monitor and is intended to represent concrete manipulatives (Moyer et al., 2002). The ability to manipulate visual representations on the computer connects the user with the real teaching and learning power of virtual manipulatives—making meaning and seeing relationships as a result of one’s own actions. Interactive, web-based visual representations of dynamic objects that represent opportunities for constructing mathematical knowledge allow users to control the actions of visual representations. The effective use of virtual manipulatives should be able to help users to connect concepts to corresponding symbolic representations (Sayeski, 2008).

Examples of virtual manipulatives websites. National Library of Virtual Manipulatives site is supported by the National Science Foundation and developed by Utah State University using Java applets for K-12 mathematics instruction. Users can choose from five content standards (NCTM, 2000) including Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. This interactive, web-based site targets interactivity for users, users' control of variable aspects, and exploration of mathematical principles and relationships (Cannon, Heal, & Wellman, 2000).

Illuminations site is part of various online activities provided by the National Council of Teachers of Mathematics. This website is designed to provide online resources that improve the teaching and learning of mathematics for K-12 grade students. The lessons and activities follow the *Principles and Standards for School Mathematics* (NCTM, 2000) and *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006).

Math Playground site includes various math games, videos and manipulatives. This site especially targets elementary and middle school students including topics of pre-algebra skills, algebraic reasoning, and rational numbers.

Research on virtual manipulatives. No research has been found on the effectiveness of virtual manipulatives in teaching mathematics, targeting students with disabilities including students with learning disabilities. However, a few studies on teaching mathematics in the general education setting provide potential suggestions and features of virtual manipulatives.

Izydorczak (2003) studied virtual manipulatives (National Library of Virtual Manipulatives; NLVM) to evaluate features of virtual manipulatives in supporting mathematical learning. Izydorczak examined the use of virtual manipulatives compared to physical manipulatives in teaching mathematics to three elementary students. In Izydorczak's study, physical manipulatives were more concrete for students and gave them more access to mathematical concepts. The researcher found that virtual manipulatives were not as concrete as physical manipulatives and led to rote understanding. Virtual manipulatives were speedy and students were able to manipulate them. Regarding the user interface aspects, virtual manipulatives, however, were inconsistent, were likely to distract users, and were found by users to be difficult to control.

Reimer and Moyer (2005) implemented a study on a small group of 19 third-graders during a two-week period on fractions with the use of virtual manipulatives. Students significantly improved in their posttests on a conceptual knowledge test, and in a relationship between conceptual knowledge and procedural knowledge. The interviews and surveys indicated that virtual manipulatives helped students learn more about fractions by providing immediate and specific feedback. Virtual manipulatives were efficient and faster to use than paper-and-pencil methods and promoted students' motivation to learn mathematics. Multiple representations of visual materials, written words, and numerical symbols through the virtual manipulatives have been found to provide scaffolding for students with LD while learning fractions by connecting visual images with abstract symbols.

Suh (2005) compared the mathematics achievement of 36 third grade students in learning the addition of fractions with unlike denominators and balancing equations in algebra, using virtual manipulatives and physical manipulatives. After two weeks of interventions, students in the virtual manipulatives instruction group outperformed students in the physical manipulatives instruction group on the posttest. According to the qualitative data in the study, the virtual manipulatives instruction better helped the learning of the procedures for the algorithm in addition of fractions than the physical manipulative instruction.

Suh and Moyer (2005) conducted a study of fifth-graders using virtual manipulatives in the classroom for learning about fractions. Results indicated that virtual manipulatives supported student's learning of fractions. Virtual manipulatives provided discovery learning, helped students make mathematical conjectures, and encouraged students to learn mathematical relationships. Virtual manipulatives also linked symbolic and iconic representations preventing a common misconception about fractions.

Steen, brooks, and Lyon (2006) examined the effect of the use of virtual manipulatives in teaching a geometry unit to 31 first grade students who were randomly assigned to the treatment or control group. The control group used their typical textbooks, using physical manipulatives. The treatment group used the same textbooks, while using virtual manipulatives. Results showed that the use of virtual manipulatives was meaningful and helped the treatment group students. The treatment group significantly outperformed the control group on the grade 2 test ($p < 0.05$).

Suh and Moyer (2007) examined the use of virtual manipulatives compared to the use of physical manipulatives in teaching 36 third grade students to identify the features of these manipulatives that enhanced students' learning. Students were assigned into two groups: one group used the Virtual Balance Scale applet from NLVM to solve linear equations; the other group used physical manipulatives, Hands-On Equations® (Borenson, 1997). Suh and Moyer found that virtual manipulatives promoted students' mathematical thinking by providing explicit links to symbolic and visual models, step-by-step support in algorithmic processes, immediate feedback and a self-checking system. The physical balance manipulative provided opportunities for invented strategies, mental mathematics, and tactile features to support students' learning.

Demir (2009) investigated 50 students who were taking a remedial mathematics course at Michigan State University and assigned them into two groups of virtual manipulatives with open-ended exploratory question group and that with structured mathematics question for four 30- to 45-minute interventions. Students using virtual manipulatives with open-ended questions showed considerably higher gains than students using virtual manipulatives with structured questions on test items requiring conceptual knowledge, yet students receiving the structured questions instruction gained higher scores on procedural and combination of conceptual and procedural knowledge items.

Haistings (2009) studied 71 first grade students and four teachers for four weeks by comparing two version of virtual manipulatives: one of Base Blocks Addition with symbolic representation and the other of Base Blocks Addition without symbolic representation. There was no statistically significant difference between the two groups

Burris (2010) conducted a qualitative study with 76 third grade students and compared two modes of instruction of place value: tactile instruction with concrete or physical manipulatives and visual instruction with computer-based or virtual manipulatives. The data showed that students' conceptualization of place value using concrete or virtual manipulatives were similar. Students receiving instruction using the virtual manipulatives could more easily construct diverse non-standard representations.

It is also important to note that many of the researchers mentioned above warn that the use of virtual manipulatives do not guarantee successful learning (Clements & McMillen 1996; Reimer & Moyer 2005). Virtual manipulatives can be used in ways that do not promote the students' conceptual understanding that they are generally purported to encourage (Clements & McMillen, 1996; Reimer & Moyer, 2005). In particular, Clements and McMillen pointed out that virtual manipulatives do not contain the meaning of the mathematical idea. Climents and McMillen also warned that virtual manipulatives can be used in a rote manner. Additionally, using appropriate virtual manipulatives for students' level of learning progress and developments is important, and computer program provides too much direct feedback (Barta, 2002). Additionally, students need to be able to use computer mouse and some computer skills as their prerequisite skills (Perl, 1990; Seo & Bryant, 2010).

General features of virtual manipulatives.

Instruction. In terms of providing materials and instructional guidance (e.g., reference), NLVM provides content-specific virtual manipulatives for the targeted mathematics concepts and skills. NLVM provided lessons according to five NCTM standards and classified them by grades K-12. Illuminations provided a clear and detailed description of instruction that is located below the lesson title of equivalent fractions.

Feedback. Virtual manipulatives provide feedback for students (Clements, 1999; Thompson, 1992) and allow students to gain a deeper understanding of complex mathematical concepts and therefore facilitate memory retention (Bouck & Flanagan, 2010). For example, the physical act of moving and adding 10 units to the number line in the one's column reinforces the mathematical principle of regrouping. Students receive feedback on incorrect responses. This feedback allows students to go back and approach the problem in a different way and provides opportunities to repeatedly apply problem-solving strategies (Bouck & Flanagan, 2010). The applet's specific and immediate feedback may also promote students' achievement on the fraction test (Suh, 2005).

Virtual manipulatives direct students to actively engage with the material, providing guiding questions, and creating multiple opportunities for success (Sayeski, 2008).

Easy accessibility. Virtual manipulative websites are free and allow easy access to everyone with a various virtual manipulative options ranging from algebra tiles, cubes, and geoboards (Bouck & Flanagan, 2010; Martin, 2007). For teachers with busy schedules in classes, these ready-to-use online resources can be efficient resources (Cannon, Heal, & Dorward, 2002; Moyer et al., 2002; Sayeski, 2008). Due to this easy accessibility, students can individually use virtual manipulatives and use them with their peers (Bouck & Flanagan, 2010).

Multiple representations. NLVM applets provide support to students with disabilities by reducing learners' cognitive load, making students directly connect between visual and numeric representations (e.g., images of fractions and the fraction symbols (Suh & Moyer, 2008). By using the computer-built-in pictorial images and symbolic notations, students could be free to focus on the mathematical connections and relationships (Kaput, 1992; Moyer-Packenham, Salkind, & Bolyard, 2008; Suh & Moyer, 2008). The critical feature of virtual manipulatives also includes dynamic images. Illumination focuses attention on important information by using different colors. For example, the website provided options selecting either "square" or "circle" and at the same time, the fractions are represented on the number line automatically.

Efficient navigation. The websites provide consistent colors, font types, font sizes, and representations throughout the entire website. For example, in NLVM, the way to display visual representations is similar even though the types are different (i.e., grid model and circle, respectively). In addition, regarding the navigation in the webpage Math Playground, users are allowed to easily navigate each lesson by selecting hyperlinked buttons (e.g., math games, word problems, logic puzzles, and math videos) in each window. Users also can click math manipulatives that link a variety of virtual manipulatives for different topics.

Summary. To sum up, web-based virtual manipulatives, which are interactive dynamic visual representations of concrete manipulatives, assist students in developing mathematical knowledge, in particular, fractions concepts and skills. Virtual manipulatives provide opportunities for constructing fractions concepts by allowing users to manipulate the visual representations on the computer screen. Virtual manipulatives also provide corrective feedback and allow students to flexibly reorganize the multiple representations on the screen. More importantly, virtual manipulatives enhance students' motivations and provide efficient ways to solve fractions problems as forms of scaffolds.

Chapter 3: Method

Fluency with fractions is a critical foundation skill of algebra (NMAP, 2008). Students are expected to understand and apply their rational number concepts to solve word problems by grade 5 (CCSS & NGA, 2010) or grade 6 (TEA, 2012). However, many students with LD, who have mathematics goals on their IEPs, have difficulties with fractions (NMAP, 2008). These students demonstrate difficulties in the conceptual understanding of fractions and they cannot understand fractions as numbers (Grobecker, 2000; Hecht & Vagi, 2010; Mazzocco & Devlin, 2008; Smith et al., 2005; Siegler et al., 2010). Furthermore, students with LD experience more of a challenge in problem-solving than their peers without LD (Cawley & Miller, 1989; L. Fuchs & D. Fuchs, 2002; Montague & Applegate, 1993) and students with LD struggle with solving word problems with fractions (Butler et al., 2003; Courey, 2006; Lambert, 1996). According to recent syntheses and reviews on teaching mathematical word problems to students with LD, cognitive and metacognitive strategies combined with the use of visual representations are effective for teaching students (Gersten, Beckmann et al., 2009; Gersten, Chard et al., 2009; Impecoven-Lind & Foegen, 2010; Maccini et al., 2007, 2008; Misquitta, 2011; Montague, 2008).

Given the significance of teaching students with LD, who have mathematics goals on their IEPs, specifically word problems with fractions and multiplication, the purpose of this study is to investigate the effects of the web-based strategic, interactive computer application (*Fun Fraction*) on the ability of middle school students with LD, who have mathematics goals on their IEPs, to solve word problems with fractions and multiplication.

Specifically, the following research questions were used in guiding this study:

1. What is the effect of a web-based strategic, interactive computer application (*Fun Fraction*) on the performance of middle school students with LD, who have mathematics IEP goals, on solving word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions?
2. How do middle school students with LD, who have mathematics IEP goals, maintain their mathematics performance when solving word problems with fractions and multiplication using a web-based strategic, interactive computer application (*Fun Fraction*) in 2 weeks following the intervention?
3. What is the perspective of middle school students with LD, who have mathematics IEP goals, on their ability to solve word problems with fractions and multiplication using a web-based strategic, interactive computer application (*Fun Fraction*)?
4. What is the perspective of middle school students with LD, who have mathematics IEP goals, on the use of cognitive and metacognitive strategies embedded in a web-based strategic, interactive computer application (*Fun Fraction*)?

Participants and Setting

Participants. Three middle school students in grades six through eight attending a private school located in a major city in the south-central region of the United States participated in this study. All participants were identified as having LD by the school and had mathematics goals on their IEPs. Also, two of the three participants (Tiffany and Alec) had been referred by the classroom teacher for behavior concerns (i.e., not staying in the classroom and not focusing on academic tasks).

Regarding classroom instruction, the business-as-usual environment showed minimal use of visual representations in learning fraction concepts and skills and lack of technology in class. Moreover, the private school was not required to take state-tests.

To find participants for the study, the researcher contacted a principal in the private school, informed him of the study, and obtained a site letter that indicated the school principal's permission to conduct the study in the school. The researcher also worked with a middle school teacher at the private school to recruit middle school students with LD in grades six through eight who had mathematics goals on their IEPs and who were interested in participating. The researcher conducted an in-person visit with the designated contact (the classroom teacher). Following this meeting, the researcher gave the teacher parent consent and student assent forms. The classroom teacher sent home the parent and student forms. The researcher was not present when the forms went home; the teacher was not representative of the research.

At the time of recruitment through the classroom teacher, students were provided with a consent form to be taken home and reviewed by a parent or legal guardian. All consent and assent forms were provided in English. Parents who permitted their child to be in the study were asked to sign their respective forms. The consent form explained the purpose of the study, procedures, and any associated risks. If a student did not return his or her consent form within 3 days, another was sent home. Assent was obtained for student participation in the study as well as for the researcher’s review of the IEP.

Previously, to be eligible to be included in this study, participants had to meet the following requirements: (a) were in grades six through eight, (b) were identified as having LD in their school or school district, (c) had mathematics goals on their IEP, (d) demonstrated a deficit in the targeted skill (i.e., earn a score below 30% accuracy on a researcher-developed screening test on word problems with fractions and multiplication). Four middle school students with LD in grades six through eight who matched all four criteria were selected for final inclusion in the study. However, one of the four students stopped participating in this project due to a time conflict and excessive absences. Finally, the rest of three middle school students with LD with mathematics IEP goals in grades six through eight included in the study. Table 3.1 provides demographic and testing information for the three participating students. Socioeconomic status (free or reduced lunch) was not collected in the school.

Table 3.1 *Demographic and Testing Profiles of Participating Students*

Variable	Tiffany	John	Alec
Age (years)	15	15	13
Grade	7	8	6
Gender	Female	Male	Male
Ethnicity	Black	Hispanic/White	Black/White
Home language	English	English	English
WIAT-III (standard score)			
Reading Comprehension	83	81	94
Math Problem Solving	72	60	56
Word Reading	77	78	95
Pseudoword Decoding	65	88	82
Numerical Operations	67	72	51
Spelling	63	75	82
Disability	LD	LD	LD
Areas of difficulty	Mathematics	Mathematics	Mathematics
Screening test (percentage)	20	20	30

Note. MLD = mathematics learning disabilities; WIAT-III = Wechsler Individual Achievement Test-Third Edition.

Setting. The study was carried out in the library (i.e., conference room), which was located next to the main office. While the study was conducted, only the researcher and participants occupied the room. The browser of *Fun Fraction* was set up in a

Windows-based PC laptop for the study. Additional equipment such as earphones, mouse, and mouse pad were provided for each student. The instructional grouping consisted of each participant independently using *Fun Fraction* to solve word problems with fractions and multiplication in a pullout instructional setting. Each intervention session lasted 30 minutes, 3 days a week (i.e., Tuesday, Wednesday, Thursday) during 6th period from 2:15 p.m. to 2:45 p.m. and included the use of the web-based strategic, interactive computer application (*Fun Fraction*) and progress monitoring. Specifically, during the intervention phase, students independently used *Fun Fraction* computer program to learn word problems with fractions and multiplication for 20 minutes and completed instructional probes for 10 minutes in each session. During the baseline and maintenance phases, students only received tests on instructional probes for 10 minutes.

Research Design

A multiple-probe single case research design across subjects was applied to assess the effects of the web-based strategic, interactive computer application (*Fun Fraction*) on the performance of middle school students with LD, who had mathematics goals on their IEPs, to solve word problems with fractions and multiplication.

With an applied behavior analysis approach, the single-case designs place a heavy value on intensively analyzing each individual case of human behaviors (Kennedy, 2005). Regarding the diverse characteristics of students with LD, single-case designs allow researchers to observe how each individual responds, comparing and contrasting with each other, one at a time (Kennedy, 2005). Single-case designs are “adaptations of interrupted time-series designs” (Kratochwill et al., 2010, p. 2) and can provide rigor as an experimental study by demonstrating functional relationships and being replicated on a larger population (Horner et al., 2005; Kennedy, 2005; Kratochwill et al., 2010).

When investigating the effects of problem-solving interventions for students with LD, many researchers applied multiple baseline designs or multiple probe designs (e.g., Case et al., 1992; Hutchinson, 1993; Jitendra et al., 1999, 2002; Joseph & Hunter, 2001; Maccini & Ruhl, 2000; Montague, 1992; Montague et al., 1993; Na, 2009; Naglieri & Johnson, 2000; Test & Ellis, 2005). In multiple baseline designs, "two or more baselines are concurrently established and the independent variable is sequentially introduced across the baselines" (Kennedy, 2005, p. 150). Multiple baseline designs do not require the withdrawal or reversal of the independent variable; thus, multiple baseline designs are especially effective in the context when the independent variable cannot be reversed after the behavior is established (Kennedy, 2005; S. Richards, Taylor, Ramasamy, & R. Richards, 1999).

As a variant of multiple baseline designs, multiple probe designs (Horner & Baer, 1978) take advantage of efficient data collection. Multiple probe designs can be implemented when extended baselines are unnecessary or impractical (Horner & Baer, 1978). In multiple probe designs, data are collected intermittently, but consistently, across all phases, especially before and after the introduction of the independent variable (i.e., *Fun Fraction*) (Kennedy, 2005).

In the multiple-probe across subjects, experimental control is demonstrated through repeated measures that can establish the prediction of the baseline's data path to the subsequent intervention phases (Kennedy, 2005). The effects of the use of *Fun Fraction* were verified by demonstrating that *Fun Fraction* promoted the correct number of word problems with fractions and multiplication on instructional probes without impacting the remaining participants' achievement during baseline, and this functional relationship was replicated across three participants (Carr, 2005). Additionally, external factors such as a sudden change of level within each phase of baseline and intervention was described in the results (Kratochwill et al., 2010).

Baseline phase. For the baseline of the intervention effect (i.e., use of *Fun Fraction*), each participant was tested on solving word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions based on their instructional probes in three or four sessions.

Intervention phase. When the first participating student's pattern on the instructional probes were consistent and predictable during baseline with three or four data points, the first participant, Tiffany, went through the intervention phase, using *Fun Fraction*. The researcher examined the functional relation of the data for an intervention effect, an increased correct percentage was observed on the instructional probes for solving word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. Each student reached mastery on word problem-solving using *Fun Fraction* for 9 or 10 days. For 7 days, each student went through 7 lessons embedded in *Fun Fraction.* They then went through 2 or 3 review sessions. Mastery was set at 80% accuracy on the instructional probes for two out of three days during the review sessions.

The effect of the intervention, *Fun Fraction,* on word problem-solving with fractions and multiplication was subsequently demonstrated for the other two students. The Intervention Phase, *Fun Fraction,* was administered to each student where a change from baseline to the intervention was observed with an increased correct percentage on the word problem tests for a minimum of three data points (Kratochwill et al., 2010). By implementing a multiple-probe design with three baseline conditions across three students, the study included three attempts to demonstrate an intervention effect with three different intervention phase repetitions, thus, showing a replication of the intervention.

Maintenance phase. Two weeks after the completion of the intervention using *Fun Fraction*, the first participant went through the maintenance phase. During the maintenance phase, the student no longer used *Fun Fraction* and received tests on the instructional probe with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions to check if the predicted intervention effect of the use of *Fun Fraction* was maintained at a similarly stable level; the student was examined if the increased correct percentage on the instructional probes was maintained. The maintenance of the effect of *Fun Fraction* was subsequently replicated across the other two students.

Independent variable. The independent variable of this study was the web-based strategic, interactive computer application (*Fun Fraction*). This intervention consisted of three main components: Multiplication Facts, Vocabulary, and Lessons. The Lessons included Lesson 1 through 7 and Review. Each session focused on one lesson for seven instructional days (See Table 3.2 for the lesson sequence). The Review provided randomly selected three word problems with fractions and multiplication for each trial. Students went through two or three Review sessions after they learned all 7 lessons.

Each of the 7 lessons included 1 modeling instruction problem (i.e., Modeling) and 4 guided instruction practice problems (i.e., Guided 1 and Guided 2). Modeling included a screen cast where students watched a video of how to solve the problem using the four cognitive strategies (i.e., Read, Restate, Represent, Answer) and metacognitive strategies of self-instruction and self-monitoring for each cognitive strategy process. The guided instruction practice had two parts: Guided 1 (1 practice problem) and Guided 2 (3 practice problems) with a gradual decrease in scaffolding. During the Guided 1 phase, all four cognitive problem-solving strategies (i.e., Read, Restate, Represent, Answer), with the use of virtual manipulatives, were taught after watching the modeling video. During the Guided 2 component, the first two strategies (Read and Restate) were faded and the other two strategies of the program, Represent and Answer with scaffolding (i.e., three-level corrective and instructional feedback; namely, students had three opportunities to try and received three different feedback based on their answers), were provided. In the program, for each cognitive strategies of Read, Restate, Represent, and Answer, students were intended to apply metacognitive strategies of self-instruction (e.g., I will read the problem. I will reread the problem if I don't understand it.) and self-monitoring (e.g., (e.g., Have I understood the problem and can now move forward?). The ultimate goal was to have students internalize the cognitive and metacognitive strategies when solving word problems (Montague et al., 2011).

Table 3.2 *Lesson Sequence*

Lesson	Operation	Problem type	Fraction type	Visual representation
1	Multiplication	Combine 1	• Multiplier: $1 < $ Whole-number ≤ 4 • Multiplicand: Proper fraction ($1 <$ denominator ≤ 4)	Rectangular area model
2	Multiplication	Combine 2	• Multiplier: $1 < $ Whole-number ≤ 4 • Multiplicand: Proper fraction ($4 <$ denominator ≤ 10)	Rectangular area model
3	Multiplication	Partition 1	• Multiplier: Proper fraction ($1 <$ denominator ≤ 10) • Multiplicand: $1 < $ Whole-number ≤ 4	Rectangular area model
4	Multiplication	Partition 2	• Multiplier: Proper fraction ($1 <$ denominator ≤ 10) • Multiplicand: Proper fraction ($1 <$ denominator ≤ 10)	Rectangular area model
5	Multiplication	Compare 1	• Multiplier: $1 < $ Whole-number ≤ 4 • Multiplicand: Proper fraction ($1 <$ denominator ≤ 10)	Rectangular area model
6	Multiplication	Compare 2	• Multiplier: Proper fraction ($1 <$ denominator ≤ 10) • Multiplicand: $1 < $ Whole-number ≤ 4	Rectangular area model
7	Multiplication	Compare 3	• Multiplier: Proper fraction ($1 <$ denominator ≤ 10) • Multiplicand: Proper fraction ($1 <$ denominator ≤ 10)	Rectangular area model
Review	Multiplication	Review lesson 1 through lesson 7 (Solve randomly selected 3 word problems)		Rectangular area model

Dependent variable. The dependent variable of this study was the percentage of correctly solved word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. The topic of multiplication of fractions aligned with middle school grade level standards such as CCSS grade 5, 5.NF3 5.NF4,5.NF5 (CCSS & NGA, 2010), TEKS grade 6 (b)(3)(B) (TEA, 2012), and Focus in grade 6 (NCTM, 2010). Participants' ability to solve word problems with fractions and multiplication was measured based on the number of word problems solved correctly on paper-and-pencil-based tests. There were three problem types: combine, partition, and compare (Taber, 2002). Additionally, for each problem type, there were three fraction types: whole-number (less than or equal to 4) multiplier and proper fraction (numerators and denominators are 1 through 10) multiplicand, proper fraction multiplier and whole-number multiplicand, and proper fraction multiplier and proper fraction multiplicand. These problem types and fraction types provided structures for lesson sequences. For each problem type with different fraction types, students were taught to manipulate the rectangular area model presented in *Fun Fraction* as a form of virtual manipulatives. The description of multiplicative situations and problem types were described in Table 3.3.

Table 3.3 *Multiplicative Situations and Problem Types*

Problem types	Fraction types: Whole-number multiplier and proper fraction multiplicand	Proper fraction multiplier and whole-number multiplicand	Proper fraction multiplier and proper fraction multiplicand
Combine	$\frac{2}{3}$ of a pie is on each tray. How many pies are on 4 trays? $4\times\frac{2}{3}$	Not applicable	Not applicable
Partition	Not applicable	John has 4 pies. He gave $\frac{2}{3}$ of them to Jane. How many pies did John give to Jane? $\frac{2}{3}\times 4$	John has $\frac{3}{4}$ of a pie. He gave $\frac{2}{3}$ of his pie to Jane. How many pies did John give to Jane? $\frac{2}{3}\times\frac{3}{4}$
Compare	John has $\frac{2}{3}$ of a pie. Jane had 4 times as many pies as John. How many pies did Jane have? $4\times\frac{2}{3}$	John has 4 pies. Jane had $\frac{2}{3}$ as many pies as John. How many pies did Jane have? $\frac{2}{3}\times 4$	John has $\frac{2}{3}$ of a pie. Jane has $\frac{3}{4}$ as many pies as John. How many pies did Jane have? $\frac{3}{4}\times\frac{2}{3}$

Development of *Fun Fraction*

Fun Fraction (**http://funfraction.org**) was designed and developed to improve the word problem-solving with fractions and multiplication performance of middle school students with LD. First, the review of studies on teaching fractions to students with LD (Bottge, 1999; Bottge et al., 2002, 2010; Butler et al., 2003; Courey, 2006; Gersten & B. Kelly, 1992; Joseph & Hunter, 2001; B. Kelly et al., 1986, 1990; Lambert, 1996; Miller

& Cooke, 1989; Test & Ellis, 2005; Woodward & Gersten, 1992) concluded the need of an intervention targeting on grade-level curriculum, in particular, multiplication of fractions for middle school students with LD. *Fun Fraction* addresses several key effective instructional components including heuristic strategies, use of visual representations, and explicit instruction. Additionally, a review of articles and books about teaching fractions and multiplication (Barnett-Clarke, Ramirez, & Coggins, 2010; Beckmann, 2011; Empson & Levi, 2011; Mack, 2001; NCTM, 2010; Petit et al., 2010; Siebert & Gaskin, 2006; Taber, 2002, 2007; Tsankova & Pjanic, 2009; H. Wu, 2010; Z. Wu, 2001) identified the features of interventions to teach multiplication word problem solving with fractions and multiplication.

In this way, by incorporating research-recommended instructional features of fraction interventions, the researcher developed *Fun Fraction* using Adobe Flash Professional CS4, Adobe Captivate 6, and Adobe Dreamweaver CS6 (i.e., HTML 5). Then the researcher bought a web hosting and a domain from www.cafe24.com.Initially, storyboards and flow charts (Appendix A) were developed and after the content of *Fun Fraction* was examined by reviewers from the departments of special education, mathematics education, and instructional technology, and by students, the curriculum and design features of *Fun Fraction* were revised several times. Through these procedures, the following key contents of *Fun Fraction* were determined: multiplication fact, vocabulary, and lessons.

Pilot-study. A pilot-study was conducted for checking the usability of *Fun Fraction*. Two middle school students with LD used *Fun Fraction* to solve word problems with fractions and multiplication and evaluated *Fun Fraction*, regarding its information, interface and interaction design features. Results showed the following feedback: (a) in Vocabulary, students wanted to change the font color and size of subtitles of each word to be distinctive color and bigger font sizes; (b) in Lessons, students want to change the color of action titles (e.g., Represent the problem using the rectangular area model) to be distinctive and suggested to change them to orange color; (c) in Represent step of four cognitive strategies (i.e., Read, Restate, Represent, Answer), students wanted to change the color of the overlapped area to be darker orange color; (d) in the video clip embedded in modeling instructional phase, students wanted the pacing of the video to be slower.

Thus, the researcher made the following changes based on students' feedback on the program: (a) in Vocabulary, changed the font color of subtitles to be darker green and font sizes from 20 pt. to 22 pt. (b) in Lessons, changed the color of action titles from blue color to orange color, distinctive from other contents; (c) in Represent step of four cognitive strategies (i.e., Read, Restate, Represent, Answer), changed the color of the overlapped area to be darker orange color; (d) made the video clip in Lessons to be slower across all 7 lesson videos. Additionally, based on participants' recommendation and preference, the overlapped area of the rectangular area model was changed to be purple color.

Features of *Fun Fraction*

The intervention, *Fun Fraction*, consisted of three components: Multiplication Facts, Vocabulary, and Lessons. Each student went through a timed-multiplication fact practice, review vocabulary, and then study (practice) lessons during each session.

Multiplication Fact. In Multiplication Fact, participants set a timer before starting to answer the multiplication facts. Whole number multiplication is a prerequisite skill needed for multiplication of fractions (NCTM, 2010); thus, participants had opportunities to practice multiplication facts in each session. Additionally, aligning with the recommendation by Gersten, Beckmann et al. (2009), participants had a 2-minute timed multiplication fact practice opportunity at the beginning of each session. The multiplication facts were presented to 10. In this way, participants were encouraged to be fluent in the retrieval of basic multiplication facts (Gersten, Beckmann et al., 2009). Figure 3.1 shows a screenshot of Multiplication Fact.

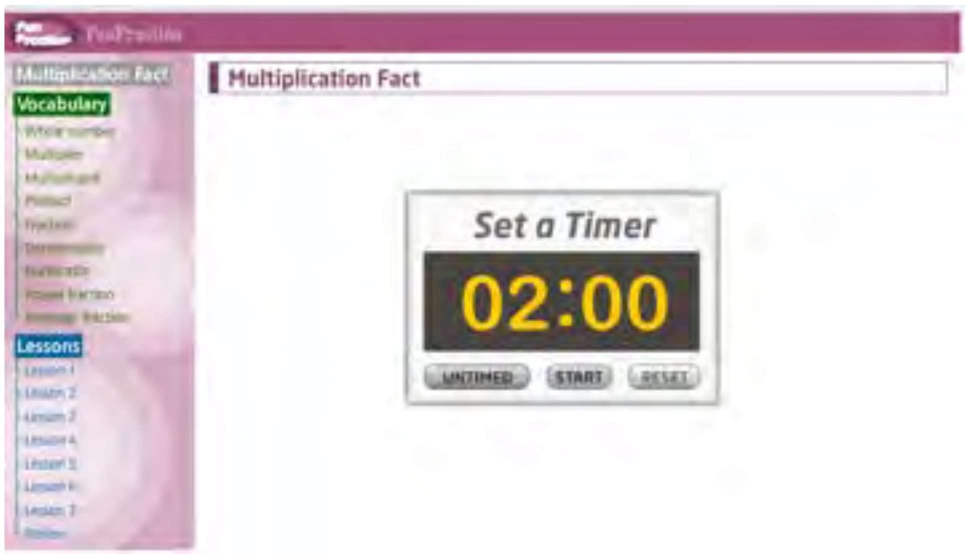


Figure 3.1. Multiplication Fact.

Vocabulary. Because mathematics has abstract and complex vocabulary and concepts, learning mathematics involves particular emphasis on the acquisition of new words (Topping, Campbell, Douglas, & Smith, 2003). In addition, because problem-solving requires the integral ability to comprehend written texts (Mestre, 1988), students need support in understanding of the concepts and representing them (Quintero, 1983). Thus, before the first session of the intervention phase, participants learned vocabulary, located to the left side of the main screen. The lists of vocabulary relevant to the multiplication of fractions, the definitions, and examples and nonexamples were excerpted from CCSS (CCSS & NGA, 2010), Siegler et al. (2010), and Texas Education Agency/University of Texas System (2011b). Vocabulary word lists included whole number, multiplier, multiplicand, product, fraction, denominator, numerator, proper fraction, and improper fraction.

During the remaining sessions, they were able to revisit lists of vocabulary by clicking the vocabulary button on the left side of the screen. The vocabulary instruction was provided in the form of a graphic organizer that helped students understand the definition of a vocabulary word and relationships of the vocabulary words using examples and nonexamples (Frayer, Frederick, & Klausmeier, 1969; Texas Education Agency/University of Texas System, 2011a). Each vocabulary word had a sound button that automatically turns on; thus, participants could go through the screen with an audio explanation from *Fun Fraction*. Figure 3.2 shows the definition, representation, examples, and nonexamples of whole number as an example of Vocabulary.

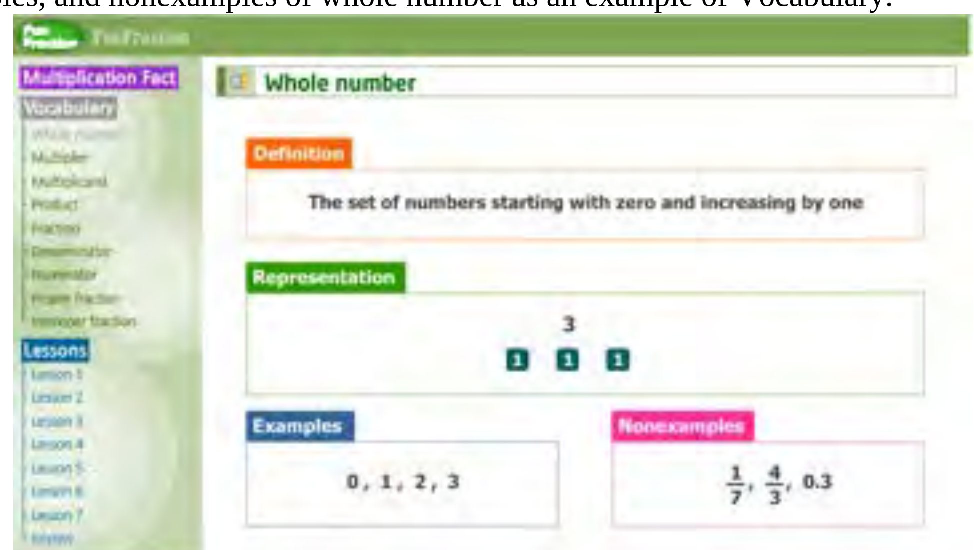


Figure 3.2. Vocabulary.

Lessons. There were 7 lessons and reviews with three problem types (combine, partition, and compare) and three fraction types: whole-number (less or equal to 4) multiplier and proper fraction (the numerator and denominator are numbers between 1 and 10) multiplicand, proper fraction multiplier and whole-number multiplicand, and proper fraction multiplier and proper fraction multiplicand. In the lessons, students were not asked to identify the problem type or fraction type because they followed the general cognitive and metacognitive strategies, implementing the use of one virtual manipulative (i.e., rectangular area model).

Participants received systematic instruction through one Modeling practice, one Guided 1 practice and three Guided 2 practices in every set of 7 lessons. The screen cast of the modeling instruction demonstrated the process of the use of *Fun Fraction* for solving word problems with fractions and multiplication for each lesson. The video clip emphasized highlighted actions with the embedded texts and colors in the video. Next, participants experienced gradually reduced scaffolded instruction through Guided 1 and Guided 2 practices. During Guided 1, students learned 4 cognitive strategies (i.e., Read, Restate, Represent, Answer) with the use of virtual manipulatives and during Guided 2, the first two strategies (Read and Restate) were faded out while two strategies (Represent and Answer) continued with the aid of embedded scaffolds with sounds and pop-up screens.

Review practices were identical with the Guided 2 practices. After 7 days of using *Fun Fraction*, participants reviewed the content of lessons 1 through 7 and solved three randomly selected word problems with fractions and multiplication each day until they arrived at mastery level of 80% correct (Scheuermann, Deshler, & Schumaker, 2009) two out of three days.

Cognitive mathematics problem solving strategies. For word problem solving, researchers recommended using cognitive strategies to solve a problem through general steps for targeted mathematics goals (Gersten, Beckmann et al., 2009; Gersten, Chard et al., 2009; Impecoven-Lind & Foegen, 2010; Seo & Bryant, 2010; Xin & Jitendra, 1999; Woodward et al., 2012). With explicit step-by-step procedures (Read, Restate, Represent, Answer*)* students were directed to solve word problems. Figure 3.3 shows how the four cognitive strategies were implemented in *Fun Fraction*.

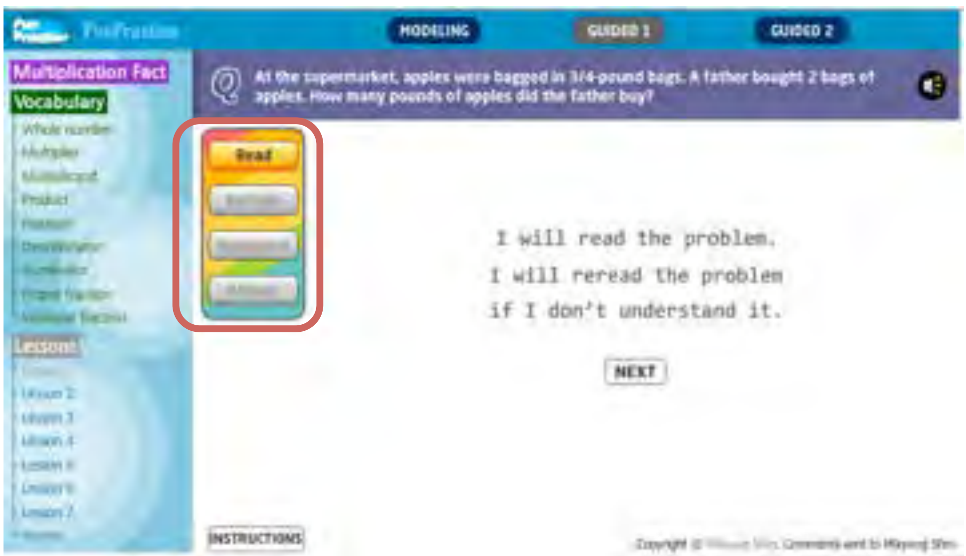


Figure 3.3. Cognitive strategies.

Metacognitive mathematics problem solving strategies. In teaching word problems, using steps to solve word problems along with monitoring with metacognitive strategies (Case et al., 1992; Impecoven-Lind & Foegen, 2010; Montague, Warger, & H. Morgan, 2000; Seo & D. Bryant, 2010) was recommended. Especially, while implementing four-step cognitive strategies, relevant metacognitive strategies (i.e., a self-coaching routine for each problem-solving step) of knowing how to solve a problem were combined. Seo and D. Bryant (2010) found that the computer-assisted four cognitive strategy instruction (i.e., Reading, Finding, Drawing, and Computing) combined with three metacognitive strategies (i.e., Do, Ask, Check) promoted the word problem-solving skills for students struggling with mathematics. The sequence of metacognitive strategies included self-instruction and self-monitoring (Montague, 1992). In *Fun Fraction*, students learned how to self-regulate their problem-solving processes. Along with the cognitive strategies of Read, Restate, Represent, and Answer, students self-instructed their action, did their action (i.e., user interface action), and self-checked their corresponding action. Specifically, the self-check metacognitive strategies were presented in the form of a pop-up screen in *Fun Fraction*. Figure 3.4 shows an example of a self-check pop-up used in the process of Read.

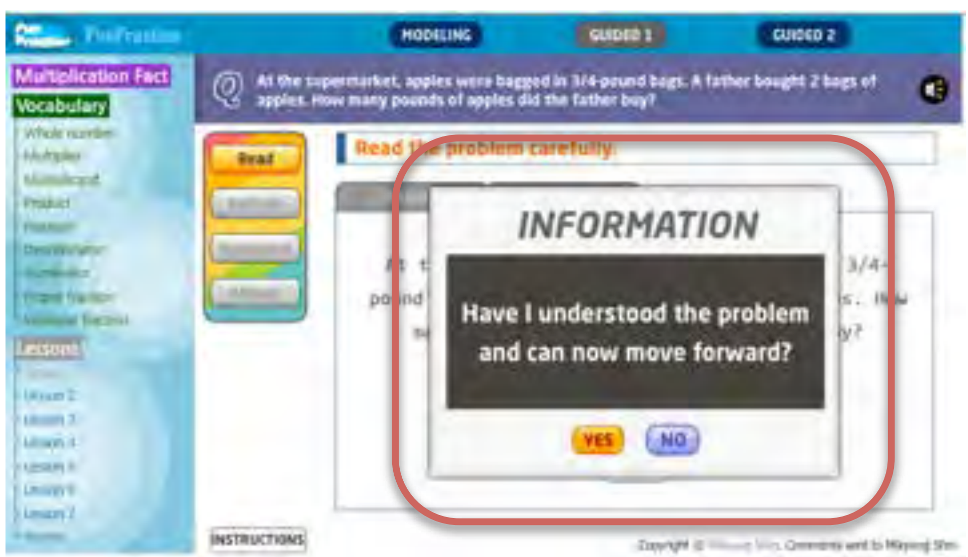


Figure 3.4. Metacognitive strategies.

Specifically, along with the four cognitive strategies of Read, Restate, Represent, and Answer, the corresponding metacognitive strategies with user interface actions are as follows:

1. Read (a) I will read the problem. I will reread the problem if I don’t understand it. (b) Read the problem carefully. (c) Have I understood the problem and can now move forward?
2. Restate (a) I will find all important information in the problem and click/highlight it. (b) Click/highlight all important information. (c) Have I found all important information in the problem and clicked/highlighted it?
3. Represent (a) I will represent the problem. (b) Represent the problem using the grid model. (c) Have I represented the problem correctly?
4. Answer (a) I will write the equation and answer it. (b) Write the equation and answer it. (c) Have I written the equation and answered it?

Table 3.4 summarizes the user interface actions and metacognitive strategies along with each subsequent cognitive strategy.

Table 3.4 *Summary of Cognitive Strategies, User Interface Actions, and Metacognitive Strategies*

Cognitive Strategies	User Interface Action	Metacognitive Strategies
Read	Read the problem carefully.	• I will read the problem. I will reread the problem if I don't understand it. • Have I understood the problem and can now move forward?
Restate	Click and highlight all importatnt information.	• I will find all important information in the problem and click and highlight it. • Have I found all important information in the problem and clicked and highlighted it?
Represent	Represent the problem using the area model.	• I will represent the problem. • Have I represented the problem correctly?
Answer	Write the equation and answer it.	• I will write the equation and answer it. • Have I written the equation and answered it?

Explicit and sequenced instruction. Lessons in *Fun Fraction* presented cognitive and metacognitive strategies in a step-by-step process by asking questions as a check for understanding (Gersten, Chard et al., 2009). With the sequenced instruction of modeling and guided practices, students had opportunities to practice and review the new concept and skills (University of Texas System/Texas Education Agency, 2011a). In Modeling, participants watched the video and looked through how to solve word problems with fractions and multiplication including two factors of whole number (less or equal to 4) and proper fractions. Modeling provided a step-by-step demonstration of how to solve word problems with fractions and multiplication with one example. When important concepts were introduced, *Fun Fraction* explicitly showed the user action (e.g. increase or decrease the denominator button) using subtitles.

In the Guided 1 and 2 components, participants used problem-solving strategies while receiving feedback on their responses. Guided 1 allowed participants to go through the same cognitive process that they learned in Modeling (i.e., Read, Restate, Represent, Answer). Guided 1 provided one word problem as a practice example. Guided 2 faded out a degree of scaffolding, focusing on Represent and Answer problems. In Guided 2, the cognitive processes of Read, Restate, Represent, Answer did not function as buttons; instead, they were represented as a prompt image, reminding participants of the processes of problem-solving (Seo, 2008). Guided 2 provided three word problems as practice examples. In the independent phase, the four cognitive strategies remained on the left hand of the screen as a prompt for the problem-solving procedures. Figure 3.5 shows the features of explicit and sequenced instruction.

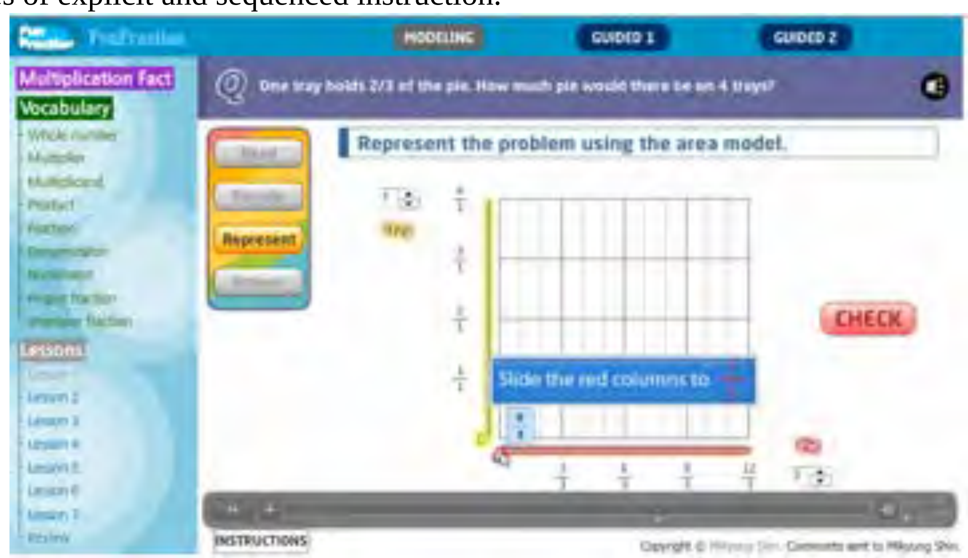


Figure 3.5. Explicit and sequenced instruction.

Virtual manipulatives. Research and practice guides in teaching mathematics to students with LD highly recommend the use of visual representations (Gersten, Beckmann et al., 2009; Gersten, Chard et al., 2009; Siegler et al., 2010; Maccini et al., 2008; Witzel et al., 2008). More recently, Woodward et al. (2012) recommended selecting visual representations appropriate for problems students solve. Studies on teaching word problem-solving for middle school students support the use of schematic diagrams to represent problems, by linking the diagrams to math sentence in order to solve problems (Jitendra et al., 1999, 2002; Na, 2009; Xin et al., 2005). For students who find difficulty in connecting various representations of mathematics concepts (Leinhardt, Zaslavsky, & Stein, 1990) including “pictorial, numerical, algebraic, graphical, and verbal models,” technology allows them to incorporate multiple representations of mathematics concepts (Garofalo & Sharp, 2003).

Rectangular area model is one of the recommended visual representations in teaching multiplication of fractions (Empson & Levi, 2011; Siegler et al., 2010; NCTM, 2010). Thus, the rectangular area model was embedded as a virtual manipulatives within the cognitive processes (i.e., Represent) in *Fun Fraction*. Virtual manipulatives apply dynamic visual representations, providing opportunities to engage in interactive learning (Bouck & Flanagan, 2010; Sayeski, 2008). Thus, the use of rectangular area model was aligned with the recommended teacher guide to implement NCTM standards such as Focus in grade 6 (NCTM, 2010) and the CCSS learning progression of fractions (Common Core Standards Writing Team, 2011). Rectangular area models were often used in the connection of prior knowledge of multiplication of whole numbers and the coordinate graph in the later grades (NCTM, 2010). Additionally, by representing the notion of unit fraction, each horizontal and vertical line had a maximum of 4 units.

Additionally, to help the visual effect, the horizontal and vertical line sliders were yellow and green respectively and the corresponding change of columns and rows match the color of the sliders. The overlapping section changed to the color purple. In the next stage of Answer, students transformed the problem represented using the rectangular area model (i.e., a schematic diagram) in Represent to equation in Answer (Xin et al., 2005). Figure 3.6 shows the example of virtual manipulatives used in *Fun Fraction.*

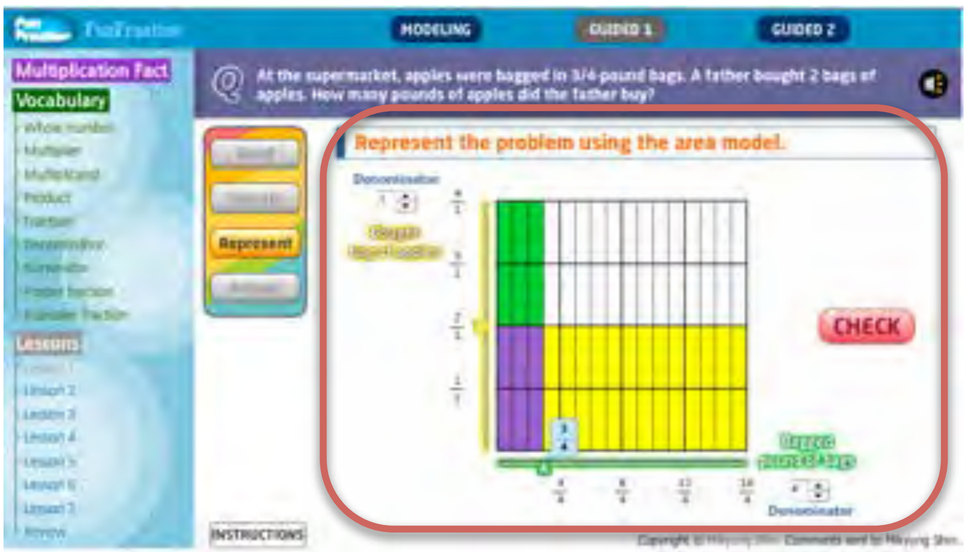


Figure 3.6. Virtual manipulatives.

Immediate feedback. Participants got immediate, corrective feedback for both correct and incorrect answers (Ellis, 2009). Research showed that while receiving computer-assisted instruction, students who received computer-generated elaboration feedback (e.g., whether their response was correct or not with an explanation) and were given the correct answer performed better than those who just received only a correct or incorrect response from a computer (Pridemore & Klein, 1991; Seo & Bryant, 2009). Thus, based on the significance of elaboration and corrective feedback, feedback was given at three levels. Students were allowed to try a maximum of three times and got feedback a maximum of three times for each step of the cognitive strategies. When students failed to find the correct answer even after three trials, they got the correct answer feedback and moved to the next step. Figure 3.7 shows an example of three-level immediate feedback in *Fun Fraction* lessons.

Figure 3.7. Immediate feedback.

Multiplication table. As scaffolding to help students to independently multiply whole number facts, a multiplication table was provided as a button form. As a tool to scaffold students' independent learning of whole number multiplication (Puntambekar & Hübscher, 2005), students were encouraged to click the multiplication table at the bottom, while writing an equation and answering it. In the table, when students clicked the two numbers, the lines were highlighted and the product of the two numbers became orange, highlighting the overlapped area. Figure 3.8 shows the Multiplication Table feature.

Figure 3.8. Multiplication table.

Measures

Screening test. To be eligible for participating in this study, students needed to answer correctly 30% or less on the researcher-developed paper-and-pencil based screening test on word problems with fractions and multiplication consisting of whole number equal to or less than unit 4 and proper fractions. Ten word problems were provided for approximately 20 minutes. Items had the same content and construct as lessons in *Fun Fraction*. The lessons, problem types (i.e., combine, partition, compare), question types (i.e., represent, equation), and guided practice numbers (i.e., Guided 1, Guided 2.1, Guided 2.2, Guided 2.3) of the 20 items during the screening test for each student were described in Appendix B.

Instructional probes. After participants learned word problem-solving with fractions and multiplication using *Fun Fraction*, they were administered a researcher-developed paper-pencil based word problem test with fractions and multiplication consisting of whole number equal to or less than unit 4 and proper fractions during each session. Five word problems were provided during each session. As a dependent measure, items in instructional probes had the same content and construct as lessons in *Fun Fraction* by including whole number units equal to or less than 4 and proper fractions. The lessons, problem types (i.e., combine, partition, compare), question types (i.e., represent, equation), guided practice numbers (i.e., Guided 1, Guided 2.1, Guided 2.2, Guided 2.3) of the five items during baseline, intervention, and maintenance phases for each student were described in Appendix B. A sample of instructional probes was provided in Appendix C.

Content validity. The researcher compiled word problem items from lessons and practice questions to construct word problem tests with fractions and multiplication. Specifically, items for instructional probes were obtained from Trends Math, Beckmann, Focus in grade 6 (NCTM, 2010), district curriculum (Envision Mathematics) and other articles on multiplication of fractions (Mack, 2001; Taber, 2007; Tsankova & Pjanic, 2009; H. Wu, 2010; Z. Wu, 2001). Additionally, the researcher systematically selected items so that items taken from Lessons 1 through 7's Modeling, Guided 1, and Guided 2 representation and equation questions could be equally distributed across alternative forms keeping the forms identical across measures.

Interrater agreement on scoring the measures. The lead researcher scored all tests across three different phases of baseline, intervention, and maintenance. Then a doctoral student independently scored more than 50% of the tests in each phase. One point was given for a correct answer. Interrater agreement was documented by dividing the total number of agreements by the total number of items, multiplying by 100%. Interrater agreement was 100% on instructional probes across the three phases.

According to Hartmann et al. (2004), minimum acceptable values of interrater agreement range from 0.80 to 0.90.

Fidelity of implementation. The lead researcher observed students' fidelity of implementation while each participant used *Fun Fraction* during the intervention phase. Across three participating students, a total of 28 intervention sessions were conducted and observed by the lead researcher. While observing students' implementation of *Fun Fraction*, fidelity checklists were used in managing consistency across different observations (See Appendix D). Regarding Lesson 1 through Lesson 7, the fidelity form consisted of 8 items that addressed Multiplication Fact practice, Modeling, Guided 1, and Guided 2. Regarding Review, the fidelity form consisted of 4 items that addressed Multiplication Fact practice and three review questions. The fidelity of implementation was calculated by dividing the total number of observed items by the total number of ratings on fidelity checklists, multiplying 100%. Fidelity of implementation was 100% for Tiffany, 94% for John, and 97% for Alec. During the nine intervention sessions, John and Alec sometimes skipped video instruction during Modeling (four times for John and two times for Alec). During the computer training, students were told that they could not skip any part of the *Fun Fraction* program.

Additionally, reliability on the fidelity of implementation was examined between the lead researcher and a doctoral student. Two doctoral students were instructed about the overall structures of the web-based *Fun Fraction* computer application and received training on how to use the checklists. The lead researcher and one of the two doctoral students observed an individual student while he or she used *Fun Fraction*, checking implementation of *Fun Fraction*. Reliability on the fidelity of implementation was assessed for at least 20% of the intervention sessions for each student. Of the total 28 intervention sessions, six were observed for reliability on the fidelity of implementation using the fidelity checklists. Reliability between the two observers was calculated by dividing the total number of agreements by the total number of ratings, multiplying 100%. For each student, reliability on the fidelity of implementation score of 100% was achieved.

Learning-related social validity and usability questionnaire. After learning word problem-solving with fractions and multiplication using *Fun Fraction*, participants independently responded to five learning-related social validity questions and evaluated the easy interface of *Fun Fraction* through nine usability questions, rating on a 5-point Likert scale (5: strongly agree, 4: agree; 3: neutral, 2: disagree, 1: strongly disagree). For the learning-related social validity questions, students were expected to think about *Fun Fraction* they used for solving word problems with fractions and multiplication; items asked if students felt the contents (i.e., Multiplication Fact, Vocabulary, Lessons, Review) of *Fun Fraction* helped them to better understand word problems with fractions and multiplication. There was an additional open-ended question about their feeling that they would use *Fun Fraction* in the future. In addition, usability questions were categorized into three aspects of design features: information, interface, and interaction. Specifically, three information criteria included appropriateness, scannability, and organization. Other three interface criteria included consistency of representations, navigation, and highlighting. The other three interactivity criteria included feedback, manipulation, and user choice. A description of each question is given in Appendix E.

Cognitive and metacognitive strategy questionnaire. After learning word problem-solving with fractions and multiplication using *Fun Fraction*, participants independently responded to two cognitive strategies-related and two metacognitive strategies-related qualitative questions. Students reflected on their experiences using cognitive and metacognitive strategies through the *Fun Fraction* program. A description of each question is given in Appendix F.

Procedure and Data Collection

Baseline phase. During the baseline phase, participating students were administered researcher-developed paper-pencil based word problem instructional probes with fractions and multiplication consisting of whole number equal to or less than unit 4 and proper fractions in three or four sessions before starting intervention sessions and introducing *Fun Fraction*; one of the tests was given immediately before the intervention began. Students were told to read the problems (or call on the examiner of they had difficulty reading) and do their best to solve them. No prompting or feedbacks on their accuracy of their work were provided to students. Students were provided with sufficient time, and no students exceeded more than 10 minutes. The order of participation was randomly selected to counterbalance a possible order effect.

Intervention phase. When a participating student's patterns and achievements on the probes were consistent and predictable during baseline with three or four data points, the student went through the intervention phase, independently using *Fun Fraction* and completing instructional probes three days a week in 30-minute sessions. Before starting Lesson 1, each student received one day of 20-minute computer training before going through Lessons using *Fun Fraction* for collecting students' data. During the computer training, participants learned the basic constructions of *Fun Fraction* and the expected routine of each session. Students learned that there were three main components of Multiplication Facts, Vocabulary, and Lessons. Students heard that every day they were expected to do 2-min multiplication facts practice on the computer before going through each lesson. Thus, students learned about how to set a 2-minute timer on the screen and learned there were two-steps of feedback on their trials of multiplication of facts. Students also heard that in each lesson they needed to go through Modeling, Guided 1, and Guided 2. Students practiced how to click buttons, navigate *Fun Fraction*, and adjust the mouse using the rectangular area model in Represent. Students also learned how to control their volume watching the video in Modeling.

Additionally, before the first Lesson 1 intervention, each student received 10-minute vocabulary instruction, while listening to vocabulary of whole number, multiplier, multiplicand, product, fraction, denominator, numerator, proper fraction, and improper fraction. Each student listened the definition, while watching representations, examples, and nonexamples on the screen. After this initial vocabulary instruction, students could review vocabulary by clicking the navigation Vocabulary button on the left side of the *Fun Fraction* screen.

After the one day of computer training and vocabulary instruction, each student started going through Lessons of *Fun Fraction*. In each lesson, students went through Modeling, Guided 1, and Guided 2 to practice how to use cognitive strategies (i.e., Read, Restate, Represent, Answer) and metacognitive strategies of self-instruction and self-monitoring for each cognitive strategy process, while implementing the rectangular area model for solving word problems with fractions and multiplication. The general instructional procedures for daily lessons through *Fun Fraction* were as follows:

1. Read the problem.
2. Identify important numbers (two quantities of multiplier and multiplicand) and see what was asked (find the multiplier)

3. When the multiplicand is less than 1, divide 1 unit into the number of the denominator of the multiplicand. When the multiplicand is greater than 1, keep the denominator as 1.
4. Increase the green color sections up to the number of the numerator of the multiplicand.
5. When the multiplier is less than 1, divide 1 unit into the number of the denominator of the multiplier. When the multiplier is greater than 1, keep the denominator as 1.
6. Increase the yellow color sections up to the number of the numerator of the multiplier.
7. Count the number of sections in unit fractions. That is the denominator of the product.
8. Count the number of purple color sections. That is the numerator of the product.

Each student was taught word problem-solving through Lessons 1 through 7 using *Fun Fraction* for 7 days. Each day, students received one lesson embedded in *Fun Fraction.* After completing the 7 lessons, students reviewed the lessons for two or three days in the Review sessions. The mastery level was equal to or above 80% correct on two out of three review days.

The effect of the use of the web-based strategic, interactive computer application (*Fun Fraction*) on word problem-solving with fractions and multiplication was subsequently demonstrated for the other two students. The functional relation was demonstrated; each student went through the intervention phase and showed an effect of using *Fun Fraction* when the previous participant demonstrated an increased correct percentage on the word problem tests with a minimum of three data points (Kratochwill et al., 2010). This procedure was continued for the third student. By implementing a multiple-probe design with three baseline conditions across three participants, the study included three attempts to demonstrate an intervention effect with three different phase repetitions. When each student was uncertain about the next steps or was distracted due to the difficulty level of the *Fun Fraction* questions, the researcher provided prompts to help them proceed in each step of cognitive strategies, in particular, Represent, on *Fun Fraction* (Seo, 2008; Snow, 2011). For example, observing students' actions, the researcher asked students why they increased or decreased the horizontal or vertical sliders or what each section of the three-colored (i.e., green, yellow, purple) rectangular area model represented (the connection between the rectangular area model and the equation solution).

Maintenance phase. Two weeks after the conclusion of the use of the web-based strategic, interactive computer application (*Fun Fraction*), the instructional probe was administered to assess the maintenance of students' ability to solve word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. During this maintenance phase, each student no longer used *Fun Fraction*. This maintenance was subsequently replicated across all three participating students. Table 3.5 summarizes the procedures for each student across baseline, intervention, and maintenance phases.

Table 3.5 *Procedures*

Phase	Task	Minute
Baseline	• Test on instructional probes for 3 or 4 days	10
Intervention	• 2-min. multiplication facts • Teach lessons 1 through 7. After 7 lessons, review for 2 or 3 days • Test on instructional probes	30
Maintenance	• Test on instructional probes in two week after completing the intervention	10

Data Analysis

The first research question addresses word problem-solving with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. Again, the population was middle school students with LD, who had mathematics IEP goals. In this study, the effect of *Fun Fraction* on students' mathematical performance of word problem-solving with fractions and multiplication was investigated.

Visual analysis. The first level of data analysis was a visual inspection of the data. Standards by the What Work Clearinghouse (Kratochwill et al., 2010) recommended using visual inspection of the data to determine the extent to which there was a functional relation between the use of *Fun Fraction* and the correct percentage on word problem tests with fractions and multiplication existed and to show the strength or magnitude of that relation. In order to describe the functional relation, six features were considered: level, trend, variability, immediacy of the effect, overlap, and consistency of the data pattern across similar phases (Horner et al., 2005; Kratochwill et al., 2010; Salzberg, Strain, & Baer, 1987).

Specifically, examining within-phase data patterns, level, trend, and variability were examined (Kratochwill et al., 2010). First, the level refers to the mean percentage of correct scores for the data; the level during the baseline phase was compared to that of the intervention phase (D. Morgan & R. Morgan, 2009). When the level of each student's accuracy percentage increases from the baseline to intervention phases, it could provide preliminary evidence that the intervention was effective (Kratochwill et al., 2010).

Next, the trend refers to the slope of the best-fitting straight line describing the data within each phase; the trend shows the decreasing or increasing prediction of data (Kratochwill et al., 2010). "The statistical technique for finding the best-fitting straight line for a set of data is called regression, and the resulting straight line is called the regression line" (p. 566, Gravetter & Wallnau, 2009). In order to find the slope of the best-fitting straight line, a graphing method with Microsoft Excel was implemented in the visually separated line graphs within each phase: baseline and intervention. If the trend increased from the baseline to the intervention phase, one can tentatively judge that there are positive results of the use of *Fun Fraction* (Ross, 2012). Cautiously, a trend in the intervention phase should not be just a continuum of trend in the baseline phase (D. Morgan & R. Morgan, 2009).

Last, the variability refers to the range or standard deviation of data around the mean (Kratochwill et al., 2010). Microsoft Excel was used to calculate the standard deviation of data in each phase of baseline and intervention. High variability within each phase reflects a failure to establish any consistency within the phase (Kratochwill et al., 2010).

Examining effects of interventions. In order to assess the effect of between-phase data patterns, immediacy of the effect, overlap, and consistency of the data pattern across similar phases were examined (Kratochwill et al., 2010). First, immediacy of the effect refers to the change in the level between the last three baseline data points and the first three intervention data points (Kratochwill et al., 2010).

Overlap refers to how much data from one phase overlaps those from the previous phase; the smaller the overlapping data between the baseline phase and the intervention phase, the greater the effect (Kratochwill et al., 2010). In this study, as nonparametric effect size measures for single-subject studies, two nonoverlap indices were used: percentage of nonoverlapping data (PND; Scruggs, Mastropieri, & Castro, 1987) and Kendall's TAU for nonoverlap of all pairs groups (Tau_{novlap}; Parker, Vannest, & Davis, 2011).

PND is the most widely used method and the easiest way to calculate nonoverlap data correlated with visual inspection (Parker, Vannest, & Davis, 2011). PND is calculated by counting the number of treatment data points that exceed the highest baseline data point and dividing by the total number of intervention points (Scruggs et al., 1987). According to the guideline by Scruggs and Mastropieri (1994), PND > 70 is interpreted as an effective effect, $50 \leq PND \leq 70$ as a questionable effect, and PND < 50 as an unreliable effect. PND, however, lacks a sampling distribution, limiting the interpretation of inference testing (Parker, Vannest, & Davis, 2011).

As an alternative to PND, Tau-U method measures data nonoverlap between two phases; Tau_{novlap} is interpreted as "nonoverlap: percentage of the nonoverlap between phases or as trendedness: percent of data showing improvement between phases" (Parker, Vannest, Davis, & Sauber, 2011, p. 291). Thus, Tau_{novlap} can be interpreted as "an asset over indices with more oblique interpretations such as Spearman Rho or least squares R or R^2" (Parker et al., 2011, p. 289), where rho = |1| is the perfect correlation between two values (Cornbleet & Shea, 1978). $\text{Tau}_{\text{novlap}}$ is derived from the Mann-Whitney U test and Kendall's Rank Correlation; because $\text{Tau}_{\text{novlap}}$ follows the "S" sampling distribution (as does the Mann-Whitney U test and Kendall's Rank Correlation), *p*-values and confidence intervals are also available (Parker, Vannest, Davis, & Sauber, 2011). Confidence intervals mean the confidence we have in an obtained effect size, and confidence intervals are recommended effect sizes by the Publication Manual of the American Psychological Association (2010). With the use of $\text{Tau}_{\text{novlap}}$, visual inspection could be fulfilled through multiple judgments about data points (Parker et al., 2011). In this study, $\text{Tau}_{\text{novlap}}$ was obtained through a web-based computer application for single-case research (Vannest, Parker, & Gonen, 2011).

Regarding the examination of the effect of between-phase data, the consistency of data in similar phases refers to how much the patterns of data are consistent within each phase: baseline and intervention; the greater consistency patterns demonstrate a better causal relationship between the baseline phase and intervention phase (Kratochwill et al., 2010). Thus, keeping consistency within each phase is a recommended feature for single-case research design.

To sum up, in order to analyze the degree to which there was an effect of the web-based strategic, interactive computer application (*Fun Fraction*) on the performance of middle school students with LD, who had mathematics IEP goals, on solving word problems with fractions and multiplication, six features of visual inspection were examined. That is, level, trend, variability, immediacy of the effect, overlap, and consistency of the data pattern across similar phases were addressed to show the functional relationship between the baseline and intervention phase (Horner et al., 2005; Kratochwill et al., 2010; Salzberg, Strain, & Baer, 1987).

Chapter 4: Results

Research Question 1

Research Question 1 examined the effect of *Fun Fraction,* a web-based strategic, interactive computer application, on the performance of middle school students with LD, who had mathematics IEP goals, on solving word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. Student's accuracy percentage scores on the researcher-developed instructional probes during the baseline, intervention, and maintenance phases and the trends of data in each baseline and intervention phase are shown in Figure 4.1. Additionally, each student's accuracy percentage scores for the problem types (i.e., combine, partition, compare), question types (i.e., represent, equation), and guided practice numbers (i.e., Guided 1, Guided 2.1, Guided 2.2, Guided 2.3) during the baseline and intervention phases are described in Appendix B.

Baseline phase. During the baseline phase, the accuracy percentage scores on the researcher-developed instructional probes showed some similar within-phase predictable patterns. Table 4.1 provides data on level, trend (i.e., slope), and variability (i.e., standard deviation and range of the data) within each baseline and intervention phase.

Tiffany. The level, namely, the mean accuracy percentage, of Tiffany's three baseline performances was 13.33%. There was a decrease in the trend of Tiffany's baseline performance. The slope of the best-fitting straight line for the baseline data was -10.00; the downward performance trend of -10.00 for Tiffany showed that the correct percentage during the baseline phase was predicted to decrease by 10.00. Tiffany's variability (i.e., fluctuation of the baseline data around the mean level of 13.33%) documented a standard deviation of 11.55 with a range of 0% to 20%. Regarding the problem types of combine, partition, and compare for each representation and equation question, Tiffany showed an accuracy percentage of 0%. She achieved a relatively higher accuracy percentage of 33% on the partition problem's representation question with multiplier as proper fractions ($1 < \text{denominator} \leq 10$) and multiplicand as a whole number ($1 < \text{Whole-number} \leq 4$) and 50% on compare problem's equation questions where multiplier was proper fractions ($1 < \text{denominator} \leq 10$) and multiplicand was a whole number ($1 < \text{Whole-number} \leq 4$).

John. Next, the level of John's baseline data was 30%. The trend of his correct percentage showed a decrease of -2.86; the correct percentage is predicted to decrease by 2.86% for each session. The average distance of baseline data from the mean level was 20 with a range of 0% to 40%. Regarding the problem types of combine, partition, and compare for each representation and equation question, John achieved relatively lower accuracy percentage scores on representation questions across the three problem types with 0% accuracy percentage on most questions; John showed relatively higher scores on equation questions for all three problems of combine, partition, and compare (range = 33% to 100%).

Alec. Regarding Alec's performance during the baseline, the level was 13.33%. The trend showed that there was no directional pattern as indicated by a slope of zero. A 23.09 standard deviation with a range of 0% to 40% demonstrated that the variability of the baseline data for Alec was the most dramatic among the three students with a standard deviation of 23.09 (range = 0% to 40%). Regarding the problem types of combine, partition, and compare under the question type of representation and equation, Alec showed the most difficulty on both combine and partition problems; he was struggling with both representation and equation questions for these problems. Alec achieved relatively higher accuracy percentage scores on compare problem's equation questions with an accuracy percentage of 50%.

Overall, of the three participating students, the level of John's baseline data showed that he had the highest baseline performance. The negative slope of best-fitting straight line of Tiffany's (-10.00) and John's (-2.86) baseline performances showed that before receiving instruction through *Fun Fraction*, their performances had been decreasing, with little variability in Tiffany's performance and a great deal of variability in Alec's baseline performance. Despite the low accuracy percentage of the three participating students on most instructional probes during the baseline on the different problem types (i.e., combine, partition, compare) and questions (i.e., representation, equation), the students achieved relatively higher accuracy percentage scores on compare problem's equation questions in general. Table 4.1 shows within-phase data patterns.

Table 4.1 *Within-Phase Data Patterns*

Student	Level			Trend		Variability *SD* (Range)	
	Baseline	Intervention	Change	Baseline	Intervention	Baseline	Intervention
Tiffany	13.33	42.00	28.67	-10.00	.61	11.55 (0~20)	22.01 (0~60)
John	30	62.22	32.22	-2.86	10.33	20 (0~40)	35.28 (0~100)
Alec	13.33	82.22	68.89	0	-.61	23.09 (0~40)	15.63 (60~100)

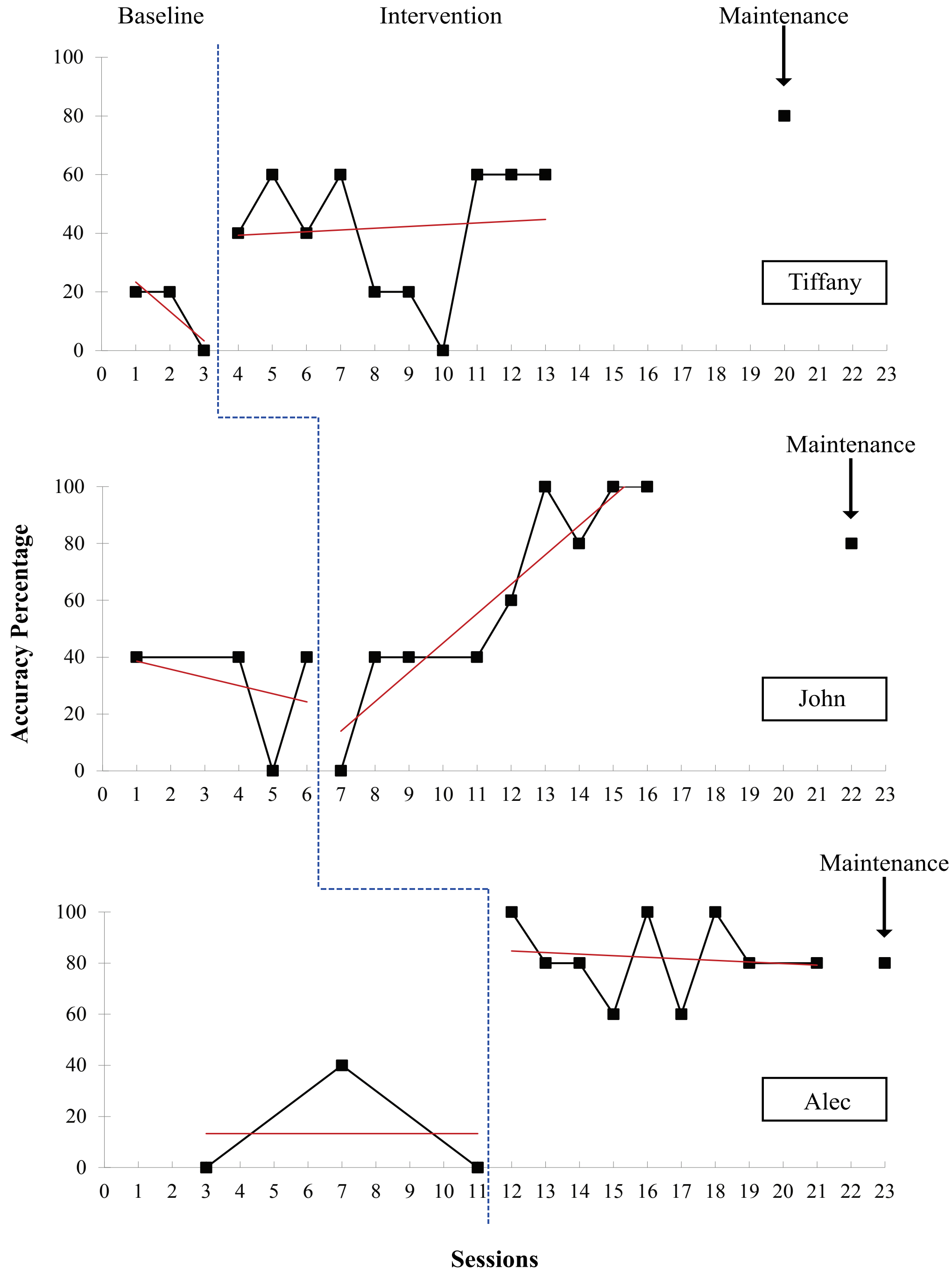
Note. Numbers show correct percentages on instructional probes.

Intervention phase. Visual inspection of Figure 4.1 shows that all three students demonstrated growth on instructional probes from their baseline to intervention phases. Each dot represents data from an instructional probe for each student (i.e., 14 for Tiffany and John; 13 for Alec). The increase in accuracy performance from the baseline to intervention phases demonstrates the effect of *Fun Fraction* on solving word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. Table 4.2 summarizes the immediacy effect, PND, and Tau_{novlap} values that were measured by comparing baseline data and intervention data.

Table 4.2 *Between-Phase (Baseline Phase Versus Intervention Phase) Data Patterns*

Student	Immediacy effect (%)	PND	Tau-U analysis			
			Tau_{novlap}	*Z*	*P*	CI 90%
Tiffany	33.34	70	.70	1.77	.08	.051<>1.349
John	0	56	.56	1.54	.12	.037<>1.148
Alec	73.34	100	1.00	2.50	.01	.341<>1.659

Note. CI = confidence interval; PND = percentage of non-overlapping data.
* *z* score means the sum of all possible pairs of data compared in a time-forward direction divided by its standard error (Parker et al., 2011).



Note. The red lines show the trend (i.e., best-fitting straight line) of data.

Figure 4.1. Accuracy percentage scores across the baseline, intervention, and maintenance phases for students.

Tiffany. Tiffany improved by 28.67% on the instructional probes from the baseline to the intervention phases with a level of 42% during the intervention phase; an immediacy effect of 33.34% was observed through the comparison of the baseline data points and the first three intervention data points. Especially, from Session 11, Tiffany received a reminder of her next actions in *Fun Fraction* with prompts on the use of the rectangular area model in connecting the representation to the concept of multiplication of fractions. The researcher sometimes asked questions regarding the student's manipulation of the rectangular area model (e.g., why she increased the section of the rectangular area model or what each color of the rectangular area model represented for).

From baseline to intervention phases, Tiffany's data showed a 70% improvement trend, which was not statistically significant (PND = 70%) (Tau_{novlap} = .70, z = 1.77, p > .05). There was 90% certainty that that the true effect size of .70 lay between 0.051 and 1.349 (CI_{90} = 0.051<>1.349) (Neyman, 1935). After receiving instruction through *Fun Fraction*, Tiffany's performance trend increased from baseline (-10.00) to intervention (.61) phases in the direction predicted by the intervention (Horner et al., 2005). The .61 for Tiffany's performance during the intervention phase indicated that the correct percentage is predicted to increase by .61% for each new session. Tiffany's intervention data fluctuated around the mean of 42% with a standard deviation of 22.01 and a range of 0% to 60%. Despite her increase during the intervention phase, Tiffany could not pass the mastery level of at least 80% correct on two of the three review days.

Problem-types and equation questions. Regarding the accuracy percentage for the problem type (i.e., combine, partition, compare) and question type (i.e., represent, equation), Tiffany's accuracy percentage scores showed some patterns within each problem type relating to the question type. For the combine problems, Tiffany achieved relatively lower scores for the represent questions (33% and 33%) than for the equation questions (60% and 50%) on instructional probes. The combine problem is similar to some of the multiplication situations with whole numbers, and this problem can be considered as a repeated-addition model (Z. Wu, 2001). For the combine problems, Tiffany often encountered errors in interpreting the one whole number (one unit); she did not add equal-sized fraction groups and automatically partitioned groups.

In addition, regarding the other problems of partition and compare types, she achieved relatively lower scores for equation questions (range = 0% to 33%) than for represent questions (range = 25% to 60%); especially, for the partition problem when both multiplier and multiplicand were proper fractions ($1 < \text{denominator} \leq 10$) and for the compare problem when multiplier was a whole number ($1 < \text{Whole-number} \leq 4$) and multiplicand was proper fractions ($1 < \text{denominator} \leq 10$), her gains were 0% for each. Her struggles with compare problems' equation questions was also observed on Session 10's instructional probes (See Figure 4.1). During Session 10, she achieved an accuracy percentage of 0% on instructional probes, which included three compare problems' equation questions as well as two partition problems' representation problems.

John. Next, there was 0% immediacy effect for John. In his first lesson, he dropped down back to 0% and the first three intervention data were 40% accuracy. The percent data showing improvement between baseline and intervention phases was 56%, which was not statistically significant (PND = 56%) (Tau_{novlap} = .56, z = 1.54, p > .05). There was 90% certainty that the true effect size of .56 lay between 0.037 and 1.148 (CI_{90} = 0.037<>1.148) (Neyman, 1935). The 56% of PND was a questionable effect (Scruggs & Mastropieri, 1994). John's accuracy percentage was 0% on the first intervention day when he did not go through all Modeling practices by skipping many parts of the video instruction. After John's initially stable accuracy percentage at about the highest baseline level, he displayed growth on instructional probes from Session 12 after receiving a reminder of his expected actions and prompts on the use of the rectangular area model in connecting the representation to the concept of multiplication of fractions. Despite no immediacy effect for John, an increase of 32.22% in the level from the baseline to the intervention phase was observed, and the level of John's nine intervention data was 62.22% during the intervention phase. John also passed the mastery level by earning 100% on two consecutive days of the three review days. The trend of his correct percentage showed a dramatic increase of 10.33 as noted in Figure 4.1; the graph sharply increased on the fifth day and did not drop back lower than 80% during the intervention phase; the correct percentage is predicted to increase by 10.33% for a new session. The average distance of intervention data from the mean of 62.22% recorded a standard deviation of 35.28 with a range of 0% to 100%.

Problem-types and equation questions. Regarding the accuracy percentage for the problem type (i.e., combine, partition, compare) and question type (i.e., represent, equation), John achieved relatively higher accuracy percentage scores than Tiffany in general. For the combine, John scored lower accuracy percentages on representation questions (33% and 67%) than on equation questions (75% and 75%); this result was consistent with Tiffany's errors in the combine problem. For compare problems, John also achieved relatively lower accuracy percentage scores on representation questions (25% and 40%) than on equation questions (50% and 100%). A sudden decrease of 20% during Session 14 occurred with compare problem's representation questions in Figure 4.1. On the contrary, for partition problems, John achieved higher accuracy percentage scores on representation questions (100% and 100%) than on equation questions (75%

and 0%). John scored 0% only for the partition problem's equation question where both multiplier and multiplicand were proper fractions ($1 < \text{denominator} \leq 10$).

Alec. Alec appeared to quickly demonstrate an experimental effect by improving sharply to 100% on his first instructional probe in the intervention phase. Alec improved the most from the baseline phase to the intervention phase by 68.89%, and the observed immediacy effect was 73.34% (the highest among the three students) by scoring a level of 82.22% during the intervention phase. Alec received a reminder of his actions in *Fun Fraction* and prompts on the use of the rectangular area model in connecting the representation to the concept of multiplication of fractions from the first intervention day (Session 12). Although there was a slight decreasing trend of -.61 (Alec's intervention data patterns are predicted to decrease by .61% for each new session), Alec was considered to have demonstrated statistically significant 100% improvement trend on the instructional probes (PND = 100%) ($\text{Tau}_{\text{novlap}}$ = 1.00, z = 2.50, p < .05). We can be 90% certain that the true effect size of 1.00 lay between 0.341 and 1.659 (CI_{90} = 0.341<>1.659) (Neyman, 1935). Noticeably, his 100% non-overlap improvement indicated that *Fun Fraction* was highly effective in regards to improving word problem solving with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. Alec also passed the mastery level by earning 80% on two of the three review days. The variability of a standard deviation of 15.63 with a range of 60% to 100% also demonstrated the least fluctuation of data among the three students.

Problem-types and equation questions. Regarding the accuracy percentage for the problem type of combine, partition, and compare, Alec achieved relatively higher scores in general than Tiffany and John. Alec's accuracy percentage scores ranged from 50% to 100%. Alec's accuracy percentage pattern for question types was clear; he achieved slightly lower accuracy percentage scores on represent questions (range = 50% to 100%) than on equation questions (67% to 100%) in general. Alec's struggled the most with combine's representation questions with multiplier as a whole number ($1 <$ whole-number ≤ 4) and multiplicand as proper fractions ($1 <$ denominator ≤ 4) (accuracy percentage = 50%) and partition problem's representation questions with multiplier as proper fraction ($1 <$ denominator ≤ 10) and multiplicand as a whole number ($1 <$ whole-number ≤ 4) (accuracy percentage = 50%). Compared to the combine and partition problems, Alec achieved relatively higher accuracy percentage scores for the compare problems on both representation and equation questions. For the compare problem's representation questions with both multiplier and multiplicand as proper fraction ($1 <$ denominator ≤ 10), Alec scored an accuracy percentage of 100%; moreover, for all the compare problem's equations questions, he gained 100% accuracy percentage scores.

In brief, all three students' levels of intervention data increased from baselines to intervention phases ranged from 56% through 100% (PND = 56% to 100%) (Tau_{novlap} = .56 to 1.00); Alec's immediacy effect was the highest followed by Tiffany's and then John's. The positive trend Tiffany and John showed after receiving instruction through *Fun Fraction*, their accuracy performances improved. Especially, John's dramatic positive trend of 10.33 demonstrated a greater effect of *Fun Fraction.* Despite the slightly decreasing trend of Alec's intervention data, there was a 68.89% level change from the baseline to the intervention phase with the least fluctuation during the intervention phase (standard deviation = 15.63), and the improvement was statistically significant.

Despite some variation in responses on instructional probes regarding students' errors and misconceptions according to the problem types of combine, partition, and compare problems and question types of representation and equation, all three students achieved relatively lower scores on representation questions than on equation questions across the three problem types. In particular, all students struggled the most with combine's representation questions, and they gained the highest accuracy percentages on compare problem's equation questions.

Research Question 2

Research Question 2 examined how middle school students with LD, who had mathematics IEP goals, maintained their mathematics performance when solving word problems with fractions and multiplication using the web-based strategic, interactive computer application (*Fun Fraction*) two weeks following the intervention. All three students maintained their intervention gains on their performance at 80% accuracy after two weeks of no instruction or practice opportunities through *Fun Fraction*.

Tiffany. Her maintenance performance of 80% was a 20% increase in performance compared to the last intervention data point (60%). Considering that Tiffany scored a level of 42% during the intervention phase, her maintenance performance of 80% was 38% higher than the level of intervention data.

John. A decrease in level occurred, with a 20% accuracy percentage score for John compared to the last intervention data point of 100%; however, compared to the intervention level of 62.22%, his maintenance improved by 17.78%.

Alec. Alec maintained the intervention gains at the same level as the last intervention data point of 80%. Alec's maintenance performance was slightly lower than the level of intervention data by 2.22%.

Problem types and equation questions. Regarding the problem types of combine, partition, and compare for each representation and equation question, students showed different responses. Tiffany and Alec made errors on representation questions;

Tiffany’s error was on the combine problem and Alec’s error was on the compare problem. John made an error on the partition problem’s equation question.

Research Question 3

Research question 3 examined the perspective of middle school students with LD, who have mathematics IEP goals, on their ability to solve word problems with fractions and multiplication using the web-based strategic, interactive computer application (*Fun Fraction*). To ascertain student information regarding the perspective on their ability to solve word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions using *Fun Fraction*, five learning-related social validity questions and nine usability questions were administered to all three participating students on their final day of participation. Learning-related questions contained experiences on Multiplication Fact, Vocabulary, Lessons, and Review. *Fun Fraction* evaluation questions were based on the design features of information (i.e., appropriateness, scannability, organization), interface (i.e., consistency of representations, navigation, highlighting), and interaction (i.e., feedback, manipulation, user choice).

Learning-related social validity questionnaire.

Multiplication Fact. Regarding the helpfulness of Multiplication Fact, the overall rating of 3.67 out of 4 indicated that participating students rated responses neutral to agree; 4 for Tiffany and Alec and 3 for John. These responses showed that the 2-minute Multiplication Fact practice was viewed fairly positive as a means to help students on multiplication facts each day before going through Lessons in *Fun Fraction*.

Vocabulary. The overall rating of 3.33 on Vocabulary also indicated that students generally held positive views on the usefulness of Vocabulary instruction; 3 for

Tiffany and John and 4 for Alec. Students went through one-time vocabulary training of about nine words (i.e., whole number, multiplier, multiplicand, product, fraction, denominator, numerator, proper fraction, improper fraction), and they could review the definitions, representations, examples, and nonexamples by clicking those vocabulary items in *Fun Fraction*.

Main learning activities. Regarding the main learning activities through Lessons, the overall rating of 3.67 on Lessons also showed participating students generally liked this module. In particular, there was variance on students' responses. Tiffany (rating = 5) and Alec (rating = 4) responded that Lessons helped them to better understand word problem-solving with fractions and multiplication. John disagreed about the helpfulness of Lessons as can be noted by his rating of 2 out of 4.

Review. Regarding the helpfulness of Review, the overall rating of 2.67 showed a slightly disagreeing attitude toward it. Students went through Review two or three days and solved three randomly selected practice questions every day after experiencing all seven days of Lessons. Only John agreed about the helpfulness of Review to review the lessons on word problem-solving with fractions and multiplication; Tiffany and John disagreed about the helpfulness of Review.

For the reflection on the learning experience of *Fun Fraction* and willingness to use *Fun Fraction* in the future, students showed different responses. Tiffany and Alec moderately liked *Fun Fraction* and were willing to use it later, too. Specifically, Tiffany said, "Maybe. I think this particular design model is very interesting for K-5. Because of the vocab and the design." Alec also stated that he felt he would use *Fun Fraction* "…because it is a good program." However, John hesitated to use *Fun Fraction* in the future. He particularly did not like the initial color of the graph (i.e., initially, the vertical color was red and the double-shaded was orange, yet due to his dislike of those colors, they were changed to green and purple, respectively). John said that he may not use it because he disliked the graph. Table 4.3 describes the summary of responses to learning-related questionnaire.

Table 4.3 *Responses to Learning-Related Questionnaire*

Student	Multiplication Fact	Vocabulary	Lessons	Review
Tiffany	4	3	5	2
John	3	3	2	2
Alec	4	4	4	4
Average	3.67	3.33	3.67	2.67

Note. This survey used a 5-point Likert scale (5: strongly agree, 4: agree; 3: neutral, 2: disagree, 1: strongly disagree).

Usability questionnaire.

Information. Regarding the information design, three aspects were considered: appropriateness, scannability, and organization. About the question whether the number of practice problems were appropriate for learning word problem-solving with fractions and multiplication, students showed neutral responses in general.

Tiffany and Alec agreed about the appropriateness of the number of practice problems, yet John disagreed with it. The overall rating of 4.50 for scannability, which asks if students could easily identify tasks, activities, and contents on the website, showed very positive attitudes. Tiffany strongly agreed and John also agreed with the scannability aspect. About the question asking if the sequence of instruction on the website was appropriate, the overall rating of 4.33 showed that students were very positive about the organization of *Fun Fraction*; both Tiffany and Alec agreed, and John strongly agreed. Table 4.4 describes the summary of responses to information questions.

Table 4.4 *Responses to Fun Fraction's Information Questionnaire*

Student	Appropriateness	Scannability	Organization
Tiffany	4	5	4
John	2	4	5
Alec	4	Not reported	4
Average	3.33	4.50	4.33

Note. This survey used a 5-point Likert scale (5: strongly agree, 4: agree; 3: neutral, 2: disagree, 1: strongly disagree).

Interface. The overall rating of 3 for all three interface design aspects (consistency of representations, navigation, and highlighting) showed a neutral attitude of the participating students. All three students showed neutral responses toward the question whether the design of the website was consistent in terms of colors, font types, and font sizes. Additionally, all of them indicated neutral attitudes for the question if the design maintained attention for important information by using appropriate colors. However, for the question that asked if each page or each window had links that were easy to navigate, Tiffany (rating = 3) and Alec (rating = 4) showed positive responses, and John disagreed with the navigation aspect. Table 4.5 describes the summary of responses to interface questions.

Table 4.5 *Responses to Fun Fraction's Interface Questionnaire*

Student	Consistency of representations	Navigation	Highlighting
Tiffany	3	3	3
John	3	2	3
Alec	3	4	3
Average	3	3	3

Note. This survey used a 5-point Likert scale (5: strongly agree, 4: agree; 3: neutral, 2: disagree, 1: strongly disagree).

Interaction. The overall ratings of interaction design regarding feedback, manipulation, and user choice indicated that interaction design aspects were the strongest design feature of the three design aspects of information, interface, and interaction. The overall ratings ranged from 4 to 4.33. Specifically, students highly agreed that feedback helped them solve mathematics problems. Tiffany and Alec rated 4 and John rated 5 for the feedback question. For the question if students were able to change rectangular area models by moving or clicking their mouse, which is a main characteristic of virtual manipulatives, all three students agreed. Last, for the question if students could easily choose lessons that they wanted to learn, Tiffany (rating = 5) and Alec (rating = 4) showed positive responses, and John showed a neutral response for the question.

Table 4.6 *Responses to Fun Fraction's Interaction Questionnaire*

Student	Feedback	Manipulation	User Choice
Tiffany	4	4	5
John	5	4	3
Alec	4	4	4
Average	4.33	4	4

Note. This survey used a 5-point Likert scale (5: strongly agree, 4: agree; 3: neutral, 2: disagree, 1: strongly disagree).

Research Question 4

Research question 4 examined the perspective of middle school students with LD, who have mathematics IEP goals, on the use of cognitive and metacognitive strategies embedded in the web-based strategic, interactive computer application (*Fun Fraction*). To ascertain student information regarding perspective on the use of cognitive and metacognitive strategies embedded in *Fun Fraction*, first, after learning word problem-solving with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions through cognitive strategies of Read, Restate, Represent, Answer, students stated that they liked the strategies and found them useful in solving word problems. John said that it helped him to understand the problem. In particularly, Tiffany and Alec pointed out that Represent was what they liked the best. Tiffany specifically stated, “Probably the most fun I had was when I ask the teacher to change the color and the very next it was changed.”

The reflection on the hardest thing that students had in learning and using Read, Restate, Represent, Answer strategies showed students’ various learning experiences. Tiffany stated that the most difficulty she experienced was when she had to redo something. When she clicked a wrong button within the Guided 1 or Guided 2 practices, she had to go through every step that she had already done; she wanted to go to the exact question that she missed, yet the problem provided a sequenced practice example. In addition, John pointed out an issue in the design aspect; he said, “It was hard seeing the letters.” Interestingly, Alec said that Represent was what he liked the best, yet the most difficult part in learning cognitive strategies in *Fun Fraction*.

Next, regarding metacognitive strategies that were embedded within *Fun Fraction*, all three students liked and felt comfortable using the strategies. Specifically, for the self-check information message (e.g., asking students if they had understood the word problem and can now move forward), John said, “It showed me what to do” and Alec stated, “Yes. I could look back manually.” For the question asking what was hard for them regarding the self-check information message, none of them found any difficulty regarding the pop-up self-check information message. Tiffany said, “Nothing that I can think of” and John said, “It’s kind of not annoying.”

Chapter 5: Discussion

The purpose of this study was to investigate the effects of the web-based strategic, interactive computer application (*Fun Fraction*) on the performance of middle school students with LD, who have mathematics goals on their IEPs. Students solved word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. In order to investigate the effects of *Fun Fraction*, a multiple-probe single case research design across subjects was applied. The effects of the use of *Fun Fraction* were investigated through the visual inspection of the functional relationship between baseline data and intervention data; this functional relation was replicated across three students.

Fun Fraction consisted of three main structures: Multiplication Fact, Vocabulary, and Lessons. The main focus of *Fun Fraction* is on Lessons, which embedded the cognitive strategies of Read, Restate, Represent, and Answer with metacognitive strategies of self-instruction and self-check of activities. In particular, within the Represent strategy, instruction on multiplication of fractions through the rectangular area model as a form of virtual manipulatives was emphasized.

The goal of *Fun Fraction* was to increase the accuracy percentage scores of word problem-solving with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions, which is considered to be a critical foundational skill for algebra (NMAP, 2008). Additionally, with the grade-level (i.e., 6^{th}) expectations for middle school students, CCSS (CCSS & NGA, 2010), NCTM's Focal Points (2006), and TEKS (TEA, 2012) emphasize instruction on multiplication of fractions for middle school students including students with LD.

Fun Fraction, a web-based strategic, interactive computer application, was based on research on teaching fractions and word problem-solving for students with LD. When teaching fractions for students with LD, concrete and visual representations, range and sequence of examples, explicit and systematic instruction, heuristic strategies, and use of real-world problems were effective instructional components that led to improvements in the fraction concepts and skills of students with LD (Bottge, 1999; Bottge et al., 2002, 2010; Bottge & Hasselbring, 1993; Butler et al., 2003; Courey, 2006; Flores & Kaylor, 2007; Gersten & Kelly, 1992; Jordan et al., 1999; Kelly et al., 1986, 1990; Lambert, 1996; Miller & Cooke, 1989; Woodward & Gersten, 1992). Additionally, when teaching word problem-solving for middle school students with LD, strategy instruction involving cognitive strategy instruction (e.g., Case et al., 1992; Joseph & Hunter, 2001; Maccini & Ruhl, 2000; Montague, 1992, 2007, 2008; Montague et al., 1993, 2011; Naglieri & Johnson, 2000) and schema-based instruction (e.g., Hutchinson, 1993; Jitendra et al., 1999, 2002; Na, 2009; Xin et al., 2005) have been found to be effective.

Fun Fraction was also based on recent IES practice guides for teaching students with LD mathematics including fractions and word problems. Gersten and Chard et al. (2009) found effective instructional components including explicit instruction, use of heuristics, using visual representations while solving problems, and range and sequence of examples for designing mathematics curriculum and instruction for students with LD. In particular, using visual representations (Gersten, Beckmann et al., 2009; Gersten, Chard et al., 2009; Siegler et al., 2010; Maccini et al., 2008; Witzel et al., 2008) that were appropriate for problem-solving was emphasized (Woodward et al., 2012). Thus, for teaching multiplication of fractions, a recommended visual representation, the rectangular area model (Empson & Levi, 2011; NCTM, 2010; Siegler et al., 2010; Taber, 2002) was selected and embedded in *Fun Fraction* as the form of virtual manipulatives which was an interactive dynamic visual representation (Martin, 2007; Martin & Lukong, 2005; Spicer, 2000). The contents of word problems with fraction and multiplication that were used in *Fun Fraction* were also taken from several recommended references from NCTM (2010) and others (Mack, 2001; Taber, 2007; Tsankova & Pjanic, 2009; H. Wu, 2010; Z. Wu, 2001).

Four research questions guided this study:

1. What is the effect of a web-based strategic, interactive computer application (*Fun Fraction*) on the performance of middle school students with LD, who have mathematics IEP goals, on solving word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions?
2. How do middle school students with LD, who have mathematics IEP goals, maintain their mathematics performance when solving word problems with fractions and multiplication using a web-based strategic, interactive computer application (*Fun Fraction*) in 2 weeks following the intervention?
3. What is the perspective of middle school students with LD, who have mathematics IEP goals, on their ability to solve word problems with fractions and multiplication using a web-based strategic, interactive computer application (*Fun Fraction*)?
4. What is the perspective of middle school students with LD, who have mathematics IEP goals, on the use of cognitive and metacognitive strategies embedded in a web-based strategic, interactive computer application (*Fun Fraction*)?

Chapter 5 discusses results in relation to the four research questions and presents conclusions drawn from the major findings. Additionally, this chapter concludes with a discussion of study's limitations, suggestions for future research, and implications for practice.

Research Question 1

Research Question 1 examined the effect of *Fun Fraction*, a web-based strategic, interactive computer application, on the performance of middle school students with LD, who had mathematics IEP goals, on solving word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. Overall the results demonstrated a gradual improvement in students' performance from baseline to intervention probes as instruction through *Fun Fraction* on word problem-solving with fractions and multiplication proceeded. All students increased correct percentages on instructional probes from the baseline phase to the intervention phase; by 28.67% for Tiffany, by 32.22% for John, and by 68.89% for Alec. In particular, Tiffany's immediacy effect of 33.34% and Alec's immediacy effect of 73.34% indicated that it was clear that each data during the baseline changed when the intervention was introduced; we can attribute the effects of this level change to the use of *Fun Fraction* rather than to any other external factors (Kratochwill et al., 2010; S. Richards et al., 1999). The percentages of data showing improvement trend between baseline and intervention phases were 70% (PND = 70%) (Tau_{novlap} = .70) for Tiffany and 56% (PND = 56%) (Tau_{novlap} = .56) for John, and these improvement trends were not statistically significant (*p*s > .05). The findings revealed that only two out of three students reached the mastery level; Tiffany could not reach the mastery level. These findings indicate that Tiffany might need more time, practice, and teacher-directed instruction in solving word problems with fractions and multiplication; instruction through the computer application alone does not work for Tiffany. The instructional effect depends on instructional components not whether it is computer-assisted instruction or teacher-directed instruction (Seo & D. Bryant, 2009). In teaching mathematics to students with LD, two additional instructional components should bed highlighted: explicit instruction through the teacher-demonstration of a step-by-step strategy and students' verbalization of their mathematical reasoning (Gersten et al., 2009). Thus, Tiffany needed more teacher demonstration of the specific cognitive and metacognitive strategies embedded in *Fun Fraction* and opportunities to verbally express her mathematical reasoning process in solving word problems with fractions and multiplication.

Due to the low accuracy percentage scores lower than the highest baseline scores during the initial intervention phase, John showed lower percent improvement trend of 56% than Tiffany's of 70%. Despite John's relatively lower percent of non-overlap, his trend (i.e., best-fitting straight line) of 10.33 during the intervention phase demonstrated that John's improvement from the baseline phase to the intervention phase was in a promising direction. Tiffany's slightly positive trend of .61 also showed that Tiffany's improvement on instructional probes during the intervention phase was moving in an expected direction. Noticeably, the accuracy percentages of Tiffany and John substantially improved in Session 11 and Session 13, respectively after they received the researcher's reminder of their expected actions in the use of *Fun Fraction* with prompts on the use of the rectangular area model in connecting the representation to the concept of multiplication of fractions.

Considering the low baseline level of Alec's accuracy performance, Alec's substantial improvement from the baseline to the intervention phase showed the highest effect of *Fun Fraction*. Despite the slightly decreasing trend of -.61 of Alec's intervention data, from baseline to intervention phases, the data showed an 100% improvement trend (PND = 100%) (Tau_{novlap} = 1.00), which was statistically significant (p < .05). Additionally, in general, regarding the problem types of combine, partition, and compare for each question type of representation and equation, students showed relatively lower accuracy percentage on the combine representation question and relatively higher accuracy percentages on the compare equation question.

Features of *Fun Fraction*.

Cognitive and metacognitive strategies. Some of the instructional features of *Fun Fraction* may be possible factors that account for the results of this study. First, the findings of this study revealed that all three students were able to apply the cognitive strategies of Read, Restate, Represent, and Answer and use metacognitive strategies by self-instructing their actions and self-checking their answers through the pop-up messages embedded in *Fun Fraction*. That is, the finding indicated that the use of *Fun Fraction,* which embedded instruction with cognitive and metacognitive strategies, enhanced students' knowledge and skills in solving word problems with fractions and multiplication of two factors of a whole number (less than or equal to 4) and proper fractions. These findings are consistent with the findings that an interactive multimedia computer-assisted instruction program, *Math Explorer*, which taught four cognitive strategies (i.e., Reading, Finding, Drawing, Computing) and three metacognitive strategies (i.e., Do, Ask, Check) was effective in one-step addition and subtraction word problem-solving skills for students with mathematics difficulties in grades 2 and 3 (Seo & D. Bryant, 2010). These findings were also consistent with previous research on cognitive strategy instruction for middle school students with LD (Case, Harris, &

Graham, 1992; Joseph & Hunter, 2001; Maccini & Ruhl, 2000; Montague, 1992, 2007, 2008; Montague et al., 2011; Montague, Applegate, & Marquard, 1993; Naglieri & Johnson, 2000).

Schematic diagram. Additionally, the rectangular area model, a schematic diagram, in the Represent strategy was used for word problem-solving with fractions and multiplication. The use of the rectangular area model was recommended by research on teaching multiplication of fractions (Empson & Levi, 2011; NCTM, 2010; Siegler et al., 2010; Taber, 2002). Students represented the three problem types of combine, partition, and compare. Across these three problem types, students manipulated the rectangular area model by considering the fraction types of whole-number multiplier and proper fraction multiplicand, proper fraction multiplier and whole-number multiplicand, and proper fraction multiplier and proper fraction multiplicand. Through the use of the rectangular area model embedded in *Fun Fraction*'s Represent step, students could demonstrate improvement on instructional probes from the base phase to the intervention phase. This finding was consistent with the previous research on schema-based instruction on teaching word problems to middle school students with LD (Hutchinson, 1993; Jitendra et al., 1999, 2002, Na, 2009; Xin et al., 2005); middle school students with LD gained improvement on their tests after schema-based instruction that included the key features of using different schema diagrams for different problem types and transforming the diagram to a mathematics sentence.

Virtual manipulatives. The form of the rectangular area model in the Represent strategy was introduced as a form of virtual manipulative: an interactive dynamic visual representation (Martin, 2007; Martin & Lukong, 2005; Spicer, 2000). The rectangular area model allowed students to manipulate the diagram to represent the multiplication of fractions by changing their denominator button and increasing or decreasing their vertical or horizontal sliders to match the numerator of each fraction. The area model was purposefully embedded as a form of virtual manipulative. Virtual manipulatives provided students with opportunities to engage in instruction through *Fun Fraction* and students could see the results of their actions of manipulation on the rectangular area model and self-correct their actions by checking their answers to the rectangular area model. Because *Fun Fraction* recognized visual changes as wrong or correct answers, it could provide correct feedback to students, helping students check their visually represented area models. Bouck and Flanagan (2010) recommended the promising use of virtual manipulatives for students with high incidence disabilities. As problem-based learning, virtual manipulatives helps students to construct their own mathematical knowledge and gain a deeper understanding of complex mathematical concepts with repeated problem-solving opportunities and feedback on their actions (C. Kelly, 2006).

Explicit and sequenced instruction. The instructional design feature of explicit and sequenced instruction embedded in Lessons' Modeling could possibly affect the positive improvement from the baseline phase to the intervention phase. As students followed all the instructional procedures of *Fun Fraction* through the explicit and sequenced instruction with Modeling, Guided 1, and Guided 2, and with the systematically faded scaffoldings provided in *Fun Fraction*, they became more adept at solving word problems with fractions and multiplication. Explicit and systematic instruction was an effective instructional component in teaching mathematics to students with LD by providing step-by-step strategies on how to solve problems, creating extensive opportunities to practice (e.g., Guided 1, Guided 2) where students could think aloud what they learned, providing corrective feedback on students' answers, and providing cumulative review (Gersten et al.; Gersten, Beckmann et al.; Gersten, Chard et al.; Jayanthi et al., 2008; NMAP). In fact, on the fist intervention day, when John did not follow all Modeling contents of Lesson 1 by skipping some of video instruction, his accuracy percentage on instructional probes was 0%. Even through John's immediacy effect was 0%, he presented a sharp increase during the intervention phase (trend = 10.33) as he proceeded, following the instructional procedures that were embedded as a form of explicit and sequenced instruction. It can be hypothesized that if each student followed all the explicit and sequenced instruction that were purposefully embedded in *Fun Fraction*, they would have been able to solve more word problems with fractions and multiplication correctly.

Students' limited mathematical and cognitive performances.

Limited conceptual understanding of multiplication of fractions. The findings of this study also indicated students' limited conceptual understanding of fractions, in particular, multiplication of fractions. As shown in Appendix B, of the representation and equation questions across three problem types of combine, partition, and compare, students struggled more with representation questions than with equation questions. Students often made errors in finding correct area models that represented the multiplication of fractions problems; all three students often could not match the correct denominator in the rectangular area model; students also lacked the concepts of the reason of partition and dividing a whole to make proper fractions; students were often challenged in understanding the meaning of the three colors presented in *Fun Fraction*'s rectangular area model; they did not understand the overlapped purple color section of the rectangular area model was the numerator of the product of the multiplication of fractions. Especially, regarding the combine representation questions, students showed the most difficulty in finding the correct area models on instructional probes. Even while using the rectangular area model in *Fun Fraction*, students showed limitations and could not distinguish if they needed to partition a whole unit number or add equally-sized fractions by increasing their sliders and keeping the denominator as 1 (e.g., $\frac{3}{1}$). These findings indicate that developing a conceptual understanding of the multiplication of fractions may require more explicit teaching time and conceptual teaching of the Represent model by the teacher along with instruction with the computer. Specifically, in teaching students with LD, providing explicit instruction through the teacher-demonstration of a step-by-step strategy and students' verbalization of their mathematical reasoning is critical (Gersten et al., 2009). Because there are different gaps among students, teachers need to encourage students to work through problems and understand reasoning behind the computer process (Snow, 2011).

These findings are consistent with previous studies on the challenges of the conceptual understanding of fractions (NMAP, 2008). Students struggling with mathematics lacked conceptual understanding of fractions symbols with part-whole relations even when they solved fractions calculation problems (Hecht & Vagi, 2010). Thus, these findings indicated the importance of conceptual knowledge of word problem-solving with fractions and multiplication through the use of the area model. In particular, while going through the four cognitive strategies of Read, Restate, Represent, and Answer, understanding of the connection between the representation of multiplication of fractions through the area model (i.e., Represent strategy in *Fun Fraction*) and the calculation of multiplication of fractions equation (i.e., Answer strategy in *Fun Fraction*) was highly important in better understanding word problem-solving with fractions and multiplication. Because conceptual and procedural knowledge influence each other's development (Baroody & Ginsburg, 1986; Hecht & Vagi, 2010; Rittle-Johnson, Siegler, & Alibali, 2001; Siegler & Stern, 1998), developing both knowledge and understanding were needed in this study.

Limited cognitive and metacognitive performances. The findings of this study demonstrated the limited cognitive and metacognitive performances of students with LD. Students showed limited working memory capacity while using *Fun Fraction* and while solving word problems with fractions and multiplication on instructional probes. Although they read a word problem and numbers presented on the screen, participants could not recall the key words and key numbers immediately thereafter. They also frequently forgot the steps of actions expected in the use of *Fun Fraction*. Regarding the limited cognitive performances, in general, all students had difficulty in focusing on the numbers carefully. In particular, John spent the least amount of time completing each lesson in *Fun Fraction* as well as on the instructional probes among the three participating students. John just quickly glanced at questions and the rectangular area model representations both on the *Fun Fraction* screen and the paper-pencil based instructional probes; then, he picked a wrong answer. However, when John spent more time for each question, he seemed to find more correct answers. On the contrary, Alec read each question carefully and spent more time than any other student. Alec made the most errors on the Multiplication fact practice, yet he used the Multiplication table for multiplying fractions. Alec was very careful in selecting answers in both representation and equation problems and showed the highest accuracy percentage on the instructional probes.

Regarding the limited metacognitive performance, motivation was also a big issue. These students had low self-confidence and low-expectation. When students made a wrong try, they showed frustration and were distracted. Thus, the researcher kept encouraging them saying "I know you can do this." Additionally, students used *Fun Fraction* under the researcher's supervision and monitoring of their performance. In the action of representing the word problem using area, students struggled the most with not knowing how to proceed. Sometimes, they were distracted by the presentation of numerator and denominator. When they received a reminder of the steps to proceed, they paid more attention. Additionally, students needed help with connecting the use of the rectangular area models and the equation problems. They often considered that the representation problem and the equation problem were not connected and showed low motivation on continuing the intervention through *Fun Fraction*. As shown in Figure 4.1, even after the initial level improvement during the intervention phase, Tiffany’s accuracy percentage decreased from Session 7 to Session 10. John also showed a stable trend and no improvement between Session 8 and Session 11. It can be hypothesized that if students were familiar with the use of rectangular area model and were able to connect the area model to the concept of multiplication of fractions, they would have paid more attention by self-correcting their mathematical misconceptions and errors and been able to solve more problems correctly on instructional probes.

Thus, in guiding students’ limited metacognitive performance for the use of the rectangular area model in connecting the representation to the concept of multiplication of fractions, the researcher provided prompts by asking questions leading students to self-check their answers so that they could connect the colored sections of the rectangular area model to the equation problem. Skylar (2008) indicated that the effective use of virtual manipulatives (i.e., rectangular area model in *Fun Fraction*) should help students connect mathematics concepts to corresponding visual representations. Tiffany and John achieved more gains from Session 11 to Session 13, when they received prompts on the use of the rectangular area model in connection to the representation of the equation. This finding indicated that the extra prompts by the researcher could have positively helped students' use of *Fun Fraction* and conceptual understanding of word problem-solving with fractions and multiplication.

Research Question 2

Research Question 2 examined how middle school students with LD, who had mathematics IEP goals, maintained their mathematics performance when solving word problems with fractions and multiplication using the web-based strategic, interactive computer application (*Fun Fraction*) in two weeks following the intervention. The three students' 80% accuracy after two weeks of no instruction showed that they maintained the intervention gains to some extent. Compared to the last intervention data point, Tiffany's accuracy percentage score even increased by 20%; Alec's was stable at the same level, and John's decreased by 20%. Compared to the level of intervention data, students' maintenance performance is explained differently; during the maintenance phase, Tiffany improved by 38%, John improved by 17.78%, and Alec showed a slight decrease of 2.22%.

Duration and instructional time. The duration and length of each session might have affected the maintenance of intervention gains through *Fun Fraction*. Tiffany received 30-minute intervention on word problems with fractions and multiplication using *Fun Fraction* 3 days per week for 10 sessions and John and Alec received the same 30-minute intervention three days per week for 9 sessions each. Three students' 9 to 10 intervention sessions met a feature of What Works Clearinghouse's Standards for a multiple baseline design by demonstrating at least 5 data points during the intervention phase (Kratochwill, 2010). Additionally, Sessions 9 and 10 included all 7 sequenced lessons and 2-3 daily reviews that were provided following the completion of lessons. During each review sessions, students practiced all three problem types of combine, partition, and compare with each representation and equation question every day with three randomly selected questions, and students might have benefited through the extra review sessions to reflect on their lessons.

Problem-types and equation questions. Regarding the problem types of combine, partition, and compare for each representation and equation question, students' word problem-solving performance during the maintenance phase was consistent with their performances during the intervention phase. Tiffany and Alec achieved relatively low scores on representation questions; Tiffany's error was on the combine problem and Alec's error was on the compare problem. On the other hand, John achieved a relatively low score on the equation question, in particular, the partition problem equation question. Considering students with LD have a substantially limited mathematical calculation and low word problem-solving performance compared to students without LD (Shin & Bryant, 2013), the three students with LD's struggling with representation and equation questions showed consistent findings. Specifically, two of the three students' difficulty in representation type of questions should be noted. Hecht, Vagi and Torgesen (2007) pointed out the significance of conceptual understanding through the representation of fractions for successful word problem-solving; rectangular area model in *Fun Fraction* can aid students in translating word problems to computation problems. Thus, students should have a better understanding of the representation questions and need to know how to interpret the rectangular area model in *Fun Fraction* to solve word problems with fractions and multiplication.

Research Question 3

Research question 3 examined the perspective of middle school students with LD, who have mathematics IEP goals, on their ability to solve word problems with fractions and multiplication using the web-based strategic, interactive computer application (*Fun Fraction*).

Learning-related social validity questionnaire. In the studies applying single-case research designs, establishing social validity, which is "the estimation of the importance, effectiveness, appropriateness, and/or satisfaction various people experience in relation to a particular intervention" (Kennedy, 2005, p. 219), is a common and recommended quality indicator for research (Horner et al., 2005). In estimating students' view on their learning through *Fun Fraction*, five learning-related questions were developed and analyzed based on suggested evaluation forms, data collection, and data analysis procedures for single-case designs (Horner et al., 2005; Kennedy, 2005; Wolf, 1978); the questionnaire used a 5-point Likert scale rating (5: strongly agree, 4: agree; 3: neutral, 2: disagree, 1: strongly disagree) for four structured questions regarding the activities embedded in *Fun Fraction* (i.e., Multiplication Fact, Vocabulary, Lessons, Review) and included one open-ended question about their willingness to use *Fun Fraction* in the future. The overall average rating of 3.67 for Multiplication Fact, 3.33 for Vocabulary, and 3.67 for Lessons showed generally positive student views on their experiences through *Fun Fraction* and found the program helpful for word problem-solving with fractions and multiplication.

Multiplication Fact. Especially, all three students gave neutral or agreeing responses for Multiplication Fact and Vocabulary activities, and the findings demonstrated the social importance of multiplication fact practices and vocabulary instruction in teaching word problem-solving with fractions and multiplication. As a prerequisite knowledge and skill for multiplication of fractions (NCTM, 2010), multiplication facts are important and students' liking the Multiplication Fact added to the value of its use in *Fun Fraction*.

Vocabulary. Additionally, students' positive evaluation of the use of Vocabulary reflected the necessity of teaching vocabulary in word problem-solving (Woodward et al., 2012). The findings also highlighted that teachers needed to consider whether students would have difficulty in understanding vocabulary and mathematical terms and provide clarification of those terms for word problem-solving.

Main learning activities. Regarding students' views on Lessons activities, Tiffany and Alec students showed agreeing and strongly agreeing responses, yet John disagreed with the helpfulness of Lessons. John's responses on the usability questionnaire showed that he was dissatisfied with the design (i.e., color of the rectangular area model) and he did not like the repetitive learning of Modeling through videos. Students' various responses regarding their learning experiences through *Fun Fraction* indicated the importance of motivation and the sources of motivation. Many students with LD have mathematics anxiety and learned helplessness; thus, students are likely to avoid mathematics tasks (Ashcraft, Krause, & Hopko, 2007; Montague, 1997b; Sideridis, 2003). Students were more engaged in the intervention when they showed better responses and understanding of word problem-solving with fractions and multiplication through *Fun Fraction*.

Review. The relatively low average rating of 2.67 for Review regarding the helpfulness of Review in reviewing lessons on word problem-solving with fractions and multiplication indicated that students felt that they already knew the contents represented through Review and did not find any additional benefits through the review activity. Additionally, it is possible that students thought that three daily review questions were not enough for them to review lessons on word problem-solving with fractions and multiplication. These findings indicate that students might need more review practice activities beyond the Guided 2 practice. For now, Review practice exercises were randomly taken from the Guided 2 practice exercises, and students might have felt that the review activities were repetitive, or that there was not enough information. Review exercises should contain both the review of lessons embedded in the *Fun Fraction* program and a review of the pre-requisite knowledge for doing multiplication and fractions.

The open-ended question about students' willingness to use *Fun Fraction* in the future also demonstrated some variance in their responses. Tiffany and Alec showed moderate positivity toward *Fun Fraction*, yet John's response showed that he did not like the graph (i.e., the color of the graph) and may not use *Fun Fraction* in the future. Students' various attitudes towards their learning experiences through *Fun Fraction* could be influenced by several factors such as their prior learning experiences about the content of multiplication of fractions and the knowledge of the computer application, virtual manipulatives (Despotakis, Palaigeorgiou, & Tsoukalas, 2007).

Usability questionnaire. After completing interventions through *Fun Fraction*, students responded to nine usability questions. Nielsen (2012) said, "Usability is a quality attribute that assesses how easy user interfaces are to use. The word "usability" also refers to methods for improving ease-of-use during the design process." Good design helps students to have good experiences leading to positive outcomes (Phyo, 2003). In design theory, information, interface, and interaction are correlated together to create a design.

Sometimes it is difficult to differentiate them because they are interrelated; as Shedroff (1999) has discussed, the three design theories are unified and the experience of one concept contributes to those of the others. In this study, usability testing was conducted to capture the profound design features of all three design theories of information (i.e., appropriateness, scannability, organization), interface (i.e., consistency of representations, navigation, highlighting), and interaction (i.e., feedback, manipulation, user choice).

The findings of the usability indicated that of the three designs of information, interface, and interaction, students response to the interaction design was the strongest. The average rating for information design ranged from 3.33 to 4.50; for interface design, all three design features' average rating was 3; interaction design's average rating ranges were 4 to 4.33. In particular, the highest rating of interaction design regarding the function of feedback and manipulation of area model in *Fun Fraction* indicated that *Fun Fraction* appropriately presented the features of computer-assisted instruction and virtual manipulatives. One of the benefits of computer-assisted instruction is providing corrective feedback to students (Pridemore & Klein, 1991; Seo & Bryant, 2009; Snow, 2011); the three-step feedback functioning as self-instructed scaffolds embedded within *Fun Fraction* with design aligns with the feature of computer-assisted instruction. Additionally, being able to change the visual representation explains the dynamic features of virtual manipulatives (Kaput, 1992; Moyer-Packenham, Salkind, & Bolyard, 2008; Suh & Moyer, 2008); thus, students' positive rating on the manipulation design feature highlights the virtual manipulative as a representative function in *Fun Fraction*.

Research Question 4

Research question 4 examined the perspective of middle school students with LD, who have mathematics IEP goals, on the use of cognitive and metacognitive strategies embedded in the web-based strategic, interactive computer application (*Fun Fraction*). First, the finding of this study showed that students liked the cognitive strategies and found them useful in solving word problems. In particular, among the four cognitive strategies of Read, Restate, Represent, and Answer, students liked Represent, through which they could manipulate the rectangular area model by changing the colored sections of each multiplier, multiplicand, and product. In the same way, students also chose Represent as the hardest strategy. Regarding metacognitive strategies that were embedded within *Fun Fraction*, the three students indicated an easy and comfortable attitude toward its use. Additionally, students liked the self-instructing and self-checking features of metacognitive strategies through which they could monitor their learning behavior and check responses.

These findings of the positive views on cognitive and metacognitive strategies are very promising. Considering that students with LD demonstrate lower cognitive and metacognitive performance than students without LD (Desoete & Roeyers, 2002; Garrett et al., 2006; Lucangeli et al., 1997; Montague & Applegate, 1993), these cognitive and metacognitive strategies embedded in *Fun Fraction* might have enhanced their limited cognitive and metacognitive performances.

Limitations

Five methodological limitations are associated with this study. First, the researcher developed instructional probes. Prior to this study, content validity for the instructional probes was established (Pierangelo & Giuliani, 2008). The researcher conducted a rigorous literature review of the recommended practices and questions for solving word problems with fractions and multiplication. Then, items were systematically taken from lessons 1 through 7’s Modeling, Guided 1, and Guided 2 representation and equation questions; items were equally distributed across alternative forms. However, instructional probes were not piloted prior to this study. Piloting the measures would have provided internal consistency reliability of measures through test-retests with alternative forms, measuring Cronbach’s Alpha; Cronbach’s Alpha is usually used to examine the consistency of results for different items for the same construct within the measure (Hunt, 2011).

Second, having only a one-time, 20-minute computer training before the actual intervention through *Fun Fraction* was not enough. Students often forgot their instructional routines during the intervention phase, and they were not familiar enough with the interface design features of *Fun Fraction* when they started to use *Fun Fraction*. Additionally, considering that the manipulation of the rectangular area model, the virtual manipulative, required some level of proficiency in using the mouse, students should have had more training in changing vertical and horizontal sliders and entering numbers in the denominator button and checking how each unit was divided prior to the intervention phase. More importantly, with the limited understanding of multiplication of fractions concepts, students failed to connect the meaning of the rectangular area model and equation questions embedded in the *Fun Fraction* program.

Third, only one short-term interval (i.e., 2 weeks) maintenance test was conducted. The short-term of 2 weeks between the completion of the intervention and maintenance test could attribute to the positive maintenance performances of the three students. Additionally, because of limited school days, it was not possible to take several maintenance tests with longer-term intervals (e.g., 4 or 6 weeks following the completion of interventions). With only a one-time maintenance test, it was not possible to confirm a specific mastery level of competence (Bloom, 1976) during the maintenance phase.

Fourth, external validity warranting that the results can be generalized beyond the experimental conditions (Kazdin, 1982) is limited in this study. Although a systematic replication across at least three participants enhances the external validity in single case design research (Horner et al., 2005; Kratochwill, et al., 2010), more students with varied level of mathematics performances should be included to generalize the results to larger populations with mathematics LD.

Finally, there was a change in the independent variable while conducting the study. Specifically, Tiffany and John received researcher's prompts about metacognitive strategies from Session 11 to Session 13 on the use of the rectangular area model in connection to the representation of the equation. Initially, the researcher did not intend to use the prompt and did not collect data on how to provide prompts to students; yet when students demonstrated limited metacognitive ability and low accuracy performance on instructional probes, it was clear that prompts were required to help students proceed with conceptual understanding in the use of *Fun Fraction*.

Suggestions for Future Research

The findings of this study provide several proposals for future research. First, the internal consistency reliability of instructional probes should be measured through test-retests with alternative forms. Additional standardized tests that include word problem-solving with fractions and multiplication concepts and skills should be included in future studies. The use of standardized tests as a dependent measure will provide more convincing results, reducing the possible manipulation of students' outcomes on the tests.

Second, students need to receive intensive computer training prior to using *Fun Fraction*. Before initiating the actual intervention through *Fun Fraction*, students need to fully experience the three design theories of information, interface, and interaction. Students must be explicitly taught the directions and instructional routines (i.e., information), be able to fluently navigate *Fun Fraction* (i.e., interface), and fully understand the function of the rectangular area model connected to the next the equation problem (i.e., interaction). Additionally, by checking students' background computer experiences (i.e., years, frequency per day, level of web surfing, and level of mouse use), recommendations for individual students for computer training can be based on each student's needs.

Third, in examining the maintenance performance of students, more than one maintenance test must be administered. Additionally, the interval between the completion of the intervention and maintenance, or that between two repeated maintenance tests, should be extended longer than 2 weeks. Additionally, a specific mastery level of competence (Bloom, 1976) should be set to compare students' performance level during the maintenance phase. Noticeably, when the study lasts for an extended time period, researchers need to be aware of the school's academic calendar in order to prevent attrition caused by middle school students' school hours and demanding assignments close to their end-of-academic-year tests.

Fourth, the effects of *Fun Fraction*, a web-based strategic, interactive computer application on the performance of middle school students with LD in solving word problems with fractions and multiplication should be further explored by including more middle school students with LD. Additionally, enhancing external validity of the generalization of the effect of *Fun Fraction* results should be used to develop randomized control trials as well as replication through single-case designs with more populations; Odom et al. (2005) pointed out that generalization of a certain practice can be encouraged with large-scale randomized classroom trials.

Fifth, when implementing a computer application as a part of content instruction, the researcher should further investigate the use of more teacher-directed instruction in developing students' conceptual understanding of the targeted mathematics topics in conjunction with technology-based instruction. When using a computer, the researcher may still need to provide prompts guiding students' metacognitive thinking to develop their self-regulatory problem-solving behaviors. As part of this research, the researcher should collect data on the occurrence of providing and systematically fading teacher prompts.

Finally, future research should examine the use of instructional design for a technology-based application. The study should examine the use of opportunities to practice cognitive and metacognitive strategies when solving word problems with fractions and multiplication. The design should be constructed in a way that helps users easily understand the features of relevant visual representations embedded in the computer. Understanding students' limited conceptual understanding of rational numbers, the researcher should study the use of more practice for students to learn and use technology, and more guidance on the use of the visual representation in connection to mathematical equations.

Implications for Practice

There were practical implications in this study. First, teachers can use *Fun Fraction*, a web-based strategic, interactive computer application, as a tool to teach word problem-solving with fractions and multiplication. *Fun Fraction* was designed to incorporate multiplication fact practices in enhancing the prerequisite concepts and skills for multiplication of fractions; vocabulary keys and instruction were included to help students having difficulty in vocabulary and mathematical terms; lessons and additional randomly review questions were included in order to apply the cognitive strategies of Read, Restate, Represent, and Answer and the metacognitive strategies of self-check and self-instruction via the use of a virtual manipulative in the form of a rectangular area model.

Second, teachers can use the cognitive and metacognitive strategies that were introduced with explicit and sequenced instruction through Modeling, Guided 1, and Guided 2 and systematically fade scaffolds, which were intended to help students initially with word problem-solving. Mathematical problem-solving is complex and requires multiple cognitive processes (Mayer, 1998; Polya, 1986) of encoding, inferring, applying, and responding (Mayer, 1998). However, many students with LD experience metacognitive difficulties; they cannot identify, monitor, and evaluate their performances as necessary for problem-solving (Desoete & Roeyers, 2002; Bannert & Mengelkamp, 2008). Additionally, considering that the students in this study expressed finding the cognitive strategies and metacognitive strategies helpful for problem-solving, the cognitive and metacognitive strategies embedded in *Fun Fraction* can function as a promising tool for teachers.

Third, teachers can use the rectangular area model embedded in the *Fun Fraction* program as a schematic diagram tool for conceptual-based problem-solving. Students in this study demonstrated limited and immature word problem-solving with fractions and multiplication while using *Fun Fraction*. Students' difficulty in translating word problems via the rectangular area model, in turn, was reflected on their relatively low accuracy performance on representation type questions compared to equation type questions. Thus, finding the correct equation for the solution does not warrant that students understand word problems rooted in the conceptual understanding of schematic diagrams which function as mental models for problem-solving (Jonassen, 2003). In helping students enhance their conceptual model-based problem-solving, students need to know how to map key elements of word problems with the assistance of schematic diagrams and link the problem representations to the equation solution (Xin et al., 2008).

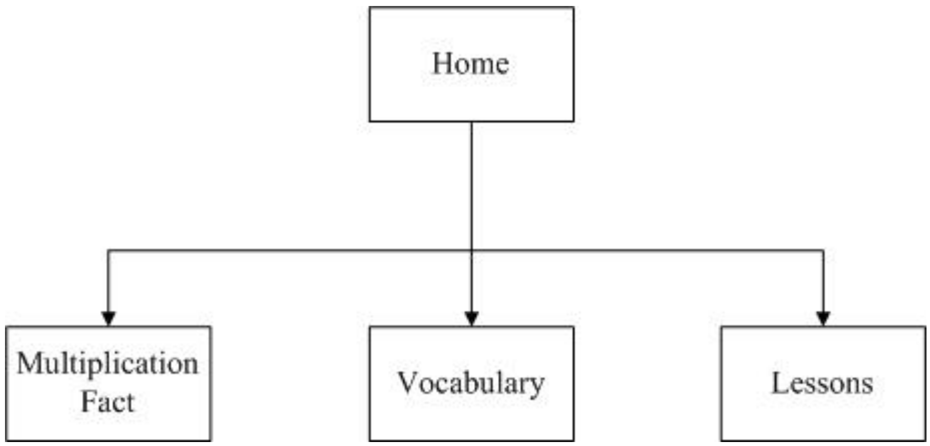
Finally, understanding the teacher's role in a classroom in which a computer application is used as a main instructional tool is essential to meet students' needs and keep students motivated and engaged. Students in this study demonstrated frequent learned helplessness when they met a challenging problem and did not attempt to connect the underlying conceptual understanding of word problem-solving via the rectangular area model to the equation problems. Additionally, students often demonstrated lack of attention and limited visual-spatial working memory; although students read the word problem correctly during Read cognitive strategy step, they forgot the key words and numbers in a following step of Represent; sometimes, students found different fraction numbers that did not match with what they had just been told (e.g., students found $\frac{4}{2}$ instead of $\frac{2}{4}$). Teachers need to monitor students if they are really engaged in the activities. Additionally, when students show some consistent misconceptions and errors, teachers need to help students to reflect on their learning activities and make sure students' actions are from their understanding not from automatic mistakes. Snow (2011) emphasized that in a classroom with the computer-assisted instruction, teachers needed to push students to engage in the work without letting students just memorize correct answers, monitor students' progress, encourage students to follow correct steps, and provide instructional adaptation as needed.

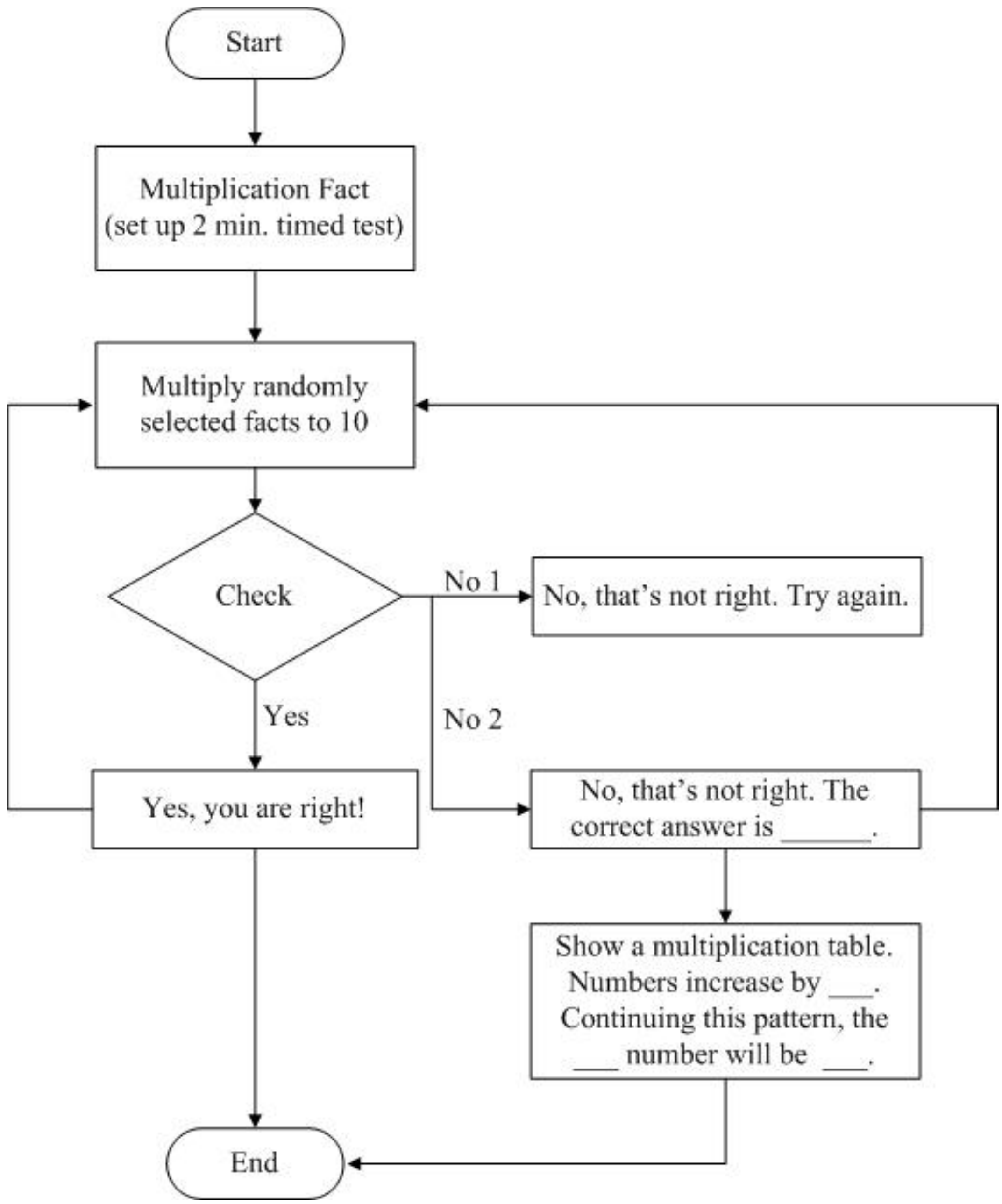
Summary

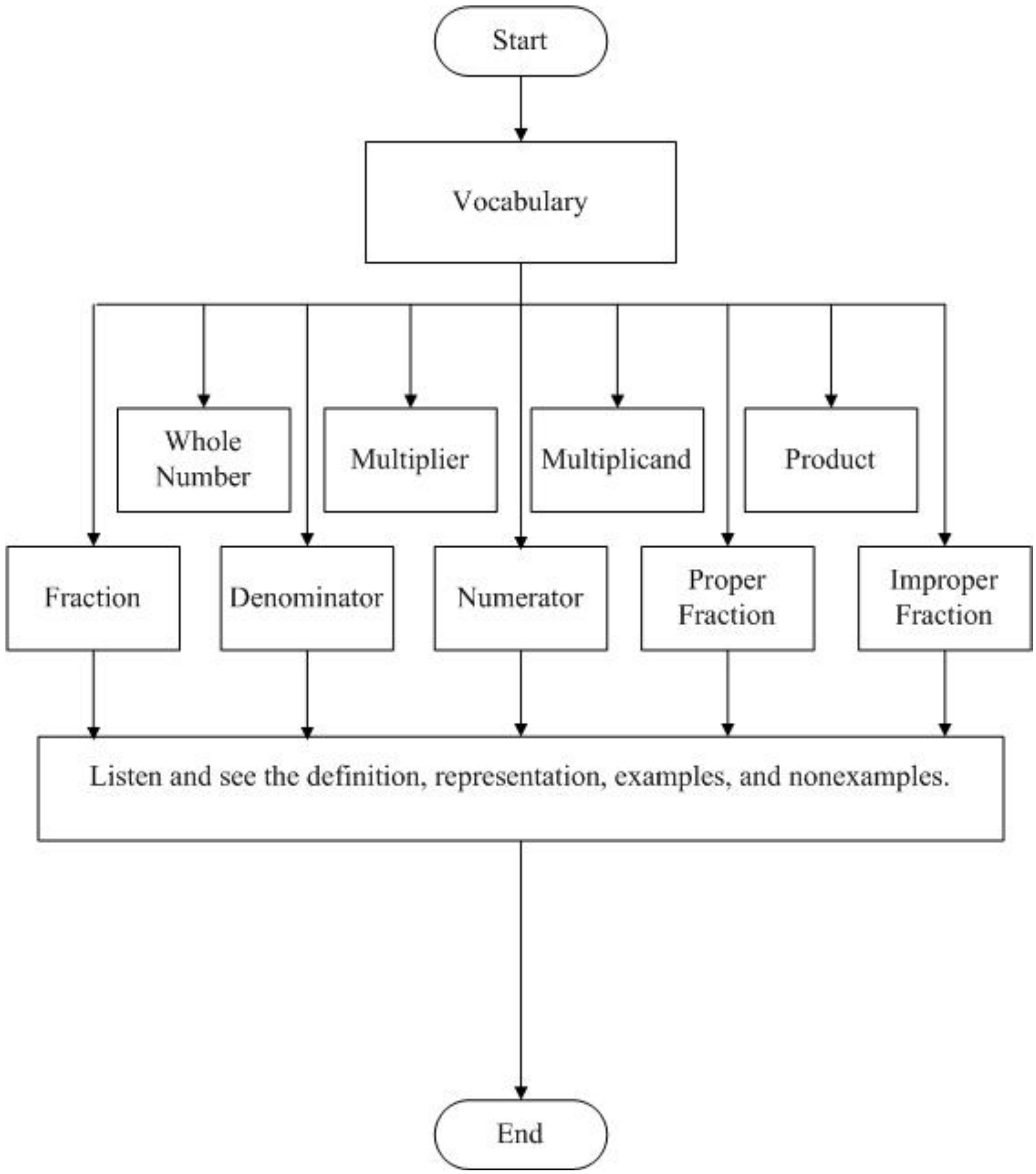
The purpose of this study was to investigate the effects of the web-based strategic, interactive computer application (*Fun Fraction*) on the performance of middle school students with LD, who have mathematics goals on their IEPs, to solve word problems with fractions and multiplication including two factors of a whole number (less than or equal to 4) and proper fractions. Given the importance of grade-level expectations (CCSS & NGA, 2010; NCTM, 2006; TEA, 2012) and word problem-solving challenges of middle school students with LD (Case et al., 1992; Hutchinson, 1993; Jitendra et al., 1999, 2002; Joseph & Hunter, 2001; Maccini & Ruhl, 2000; Montague, 1992, 2007, 2008; Montague et al., 1993, 2011; Na, 2009; Naglieri & Johnson, 2000; Xin et al., 2005), teaching word problems with fractions and multiplication to middle school with LD using the web-based strategic, interactive computer application using virtual manipulatives warrants research.

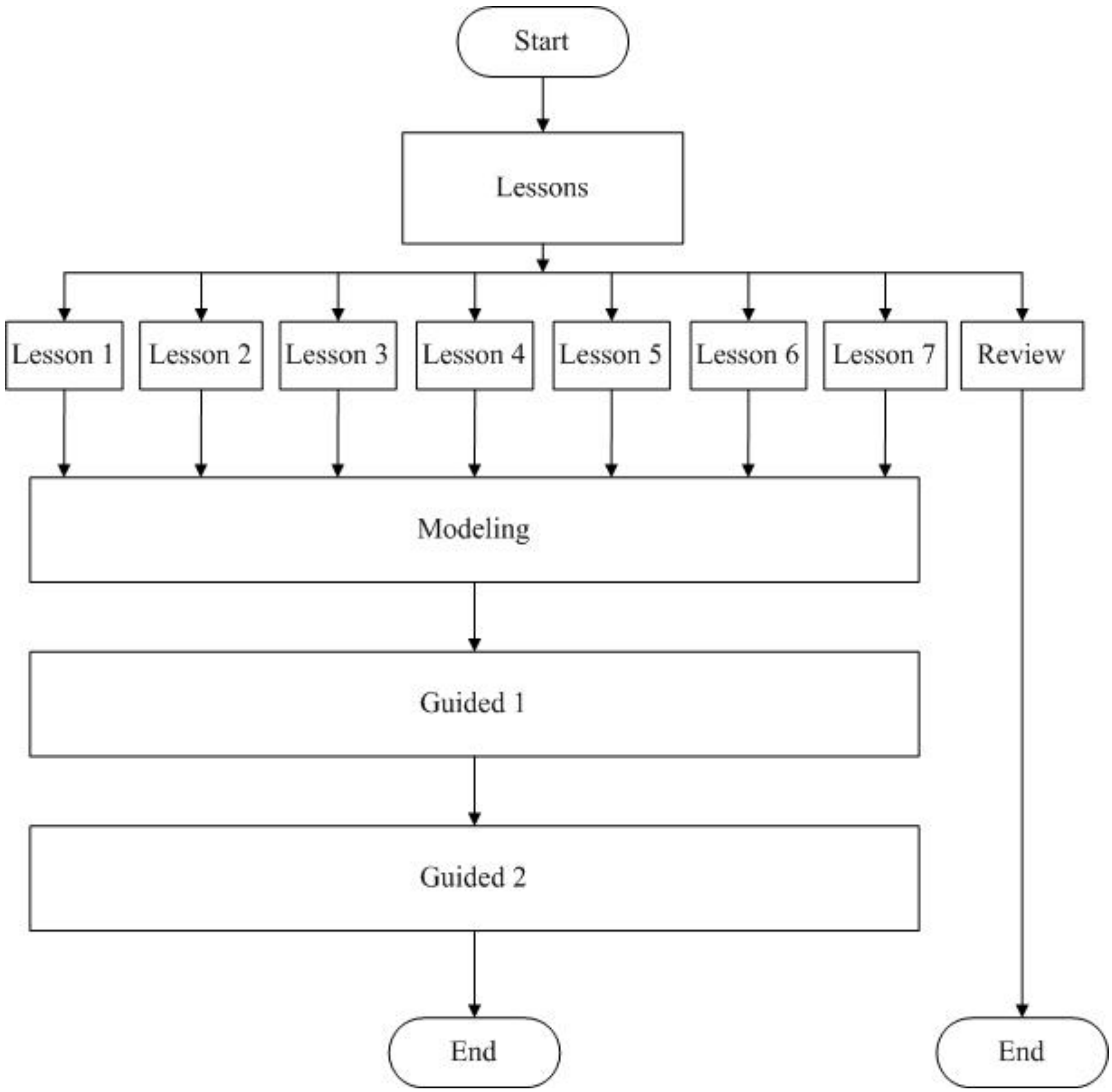
The results of the study revealed that students achieved higher accuracy percentage scores on instructional probes during the intervention phase and maintenance phase compared to the baseline phase. Although two of the three students reached the mastery level during the intervention phase, the change of level from the baseline to the intervention phase supports the effect of instruction through *Fun Fraction* on word problems with fractions and multiplication. Additionally, students' difficulty in the representation type question (i.e., combine representation question) demonstrated students had a relatively limited conceptual understanding of the multiplication of fractions that could be pretended through the competence of the representation question on instructional probes. The findings of the study suggested that students' understanding of the mathematical concept with the assistance of the rectangular area model and solving equation problems are connected and help each other in solving word problems with fractions and multiplication.

Appendix A: Flow Chart of *Fun Fraction*

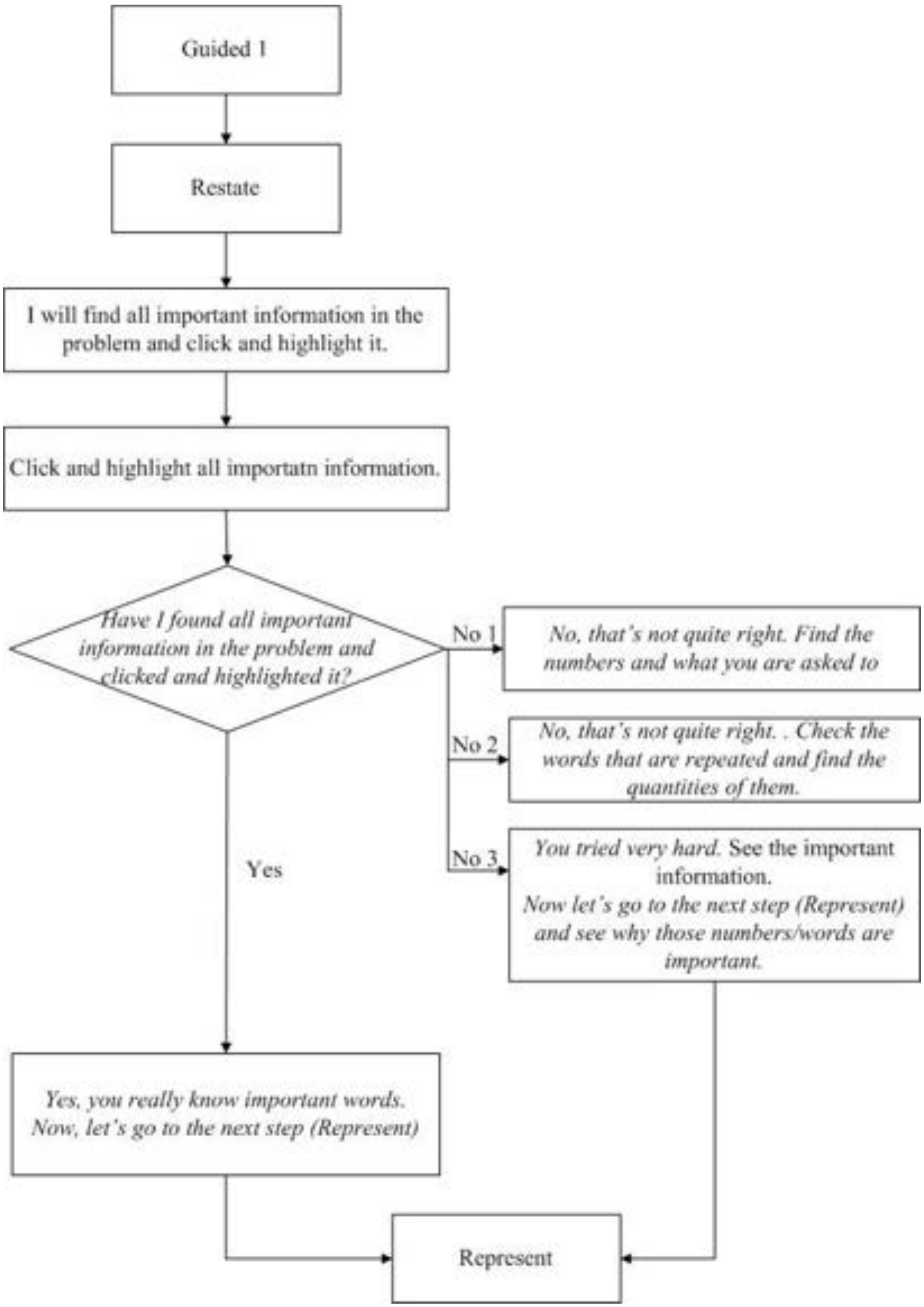


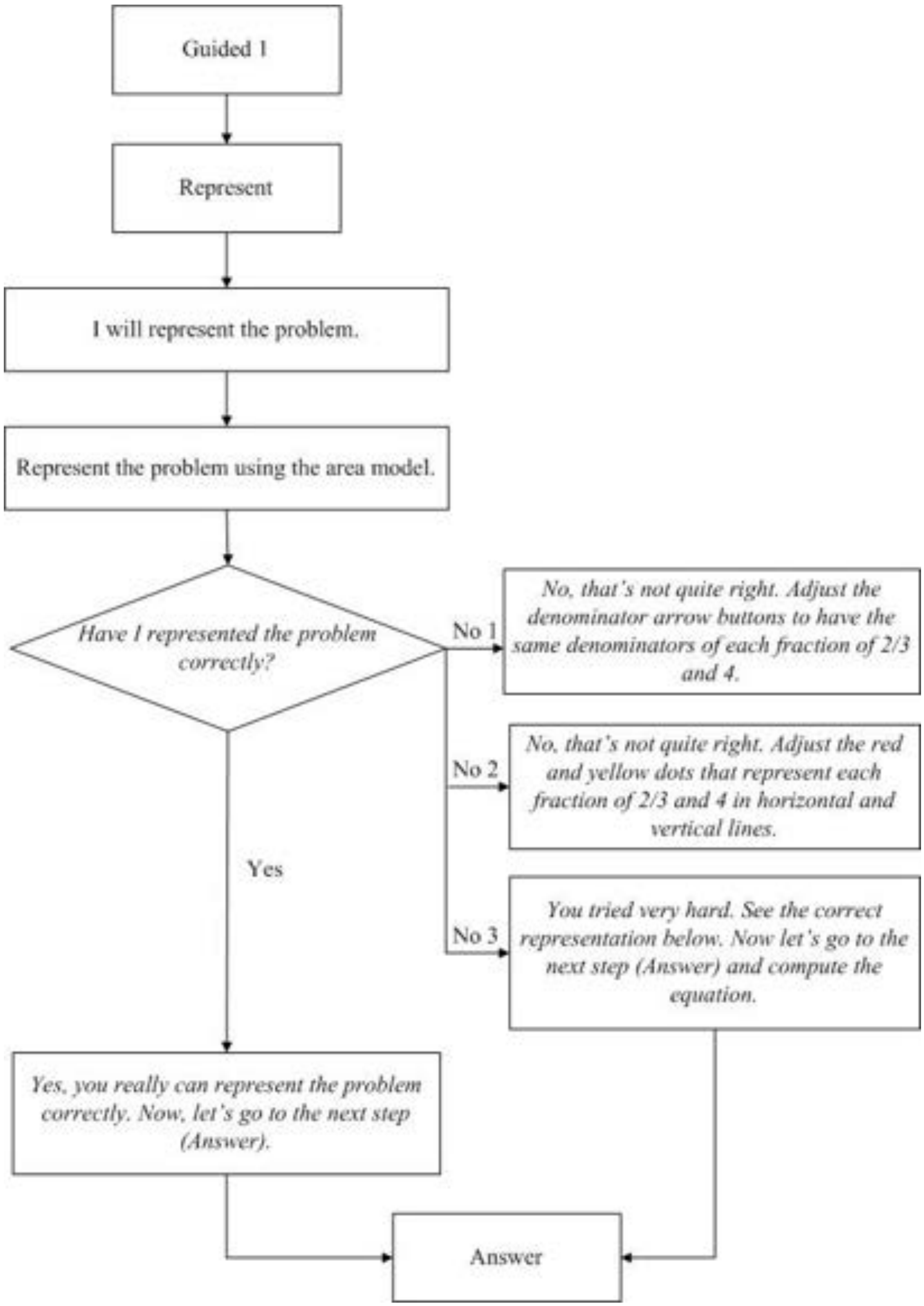

Start
Multiplication Fact
(set up 2 min. timed test)
Multiply randomly
selected facts to 10
Check
No 1
No, that's not right. Try again.
Yes
No 2
Yes, you are right!
No, that's not right. The
correct answer is ______.
Show a multiplication table.
Numbers increase by ___.
Continuing this pattern, the
___ number will be ___.
End

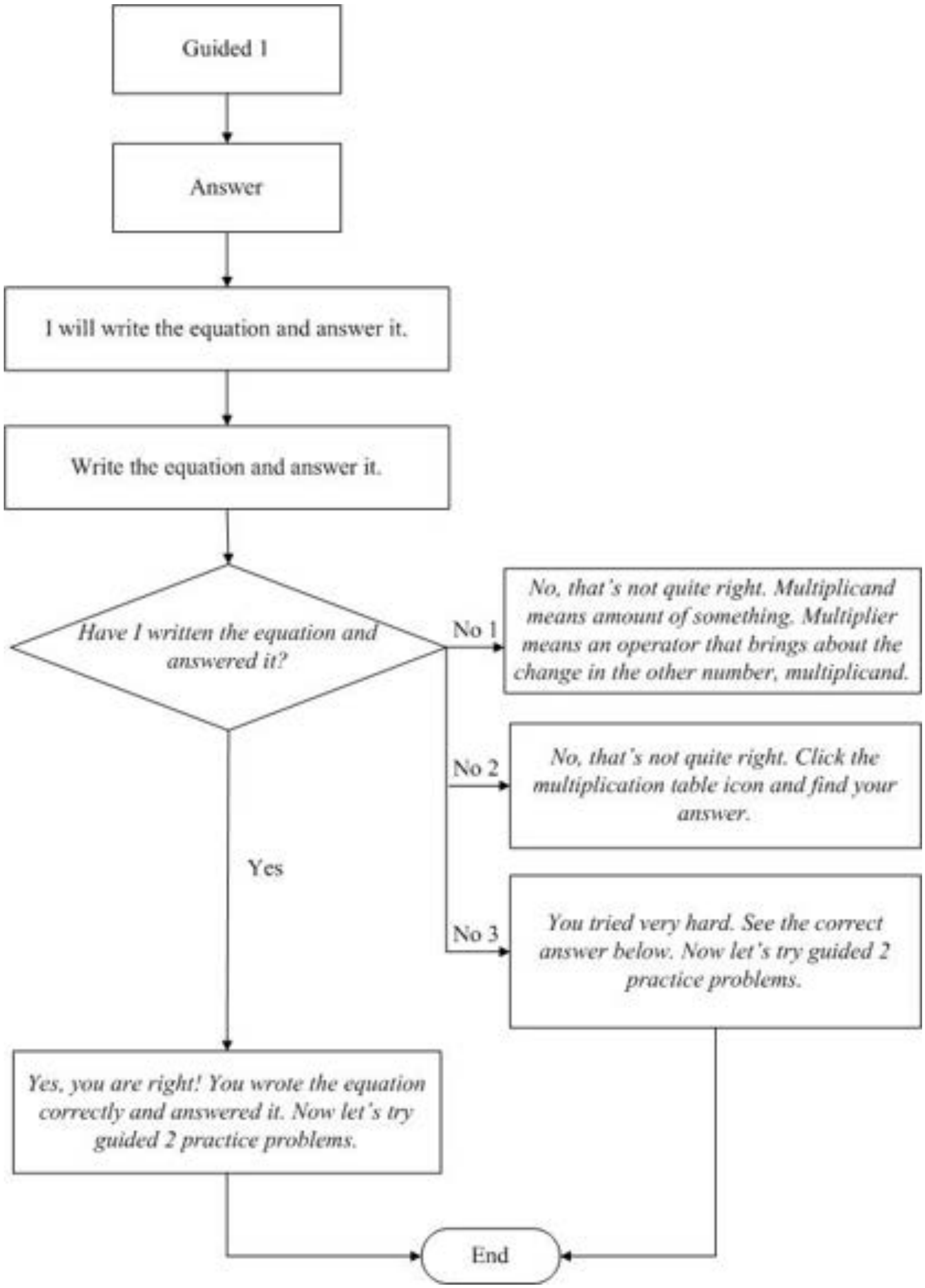

Start
Vocabulary
Whole Number
Multiplier
Multiplicand
Product
Fraction
Denominator
Numerator
Proper Fraction
Improper Fraction
Listen and see the definition, representation, examples, and nonexamples.
End

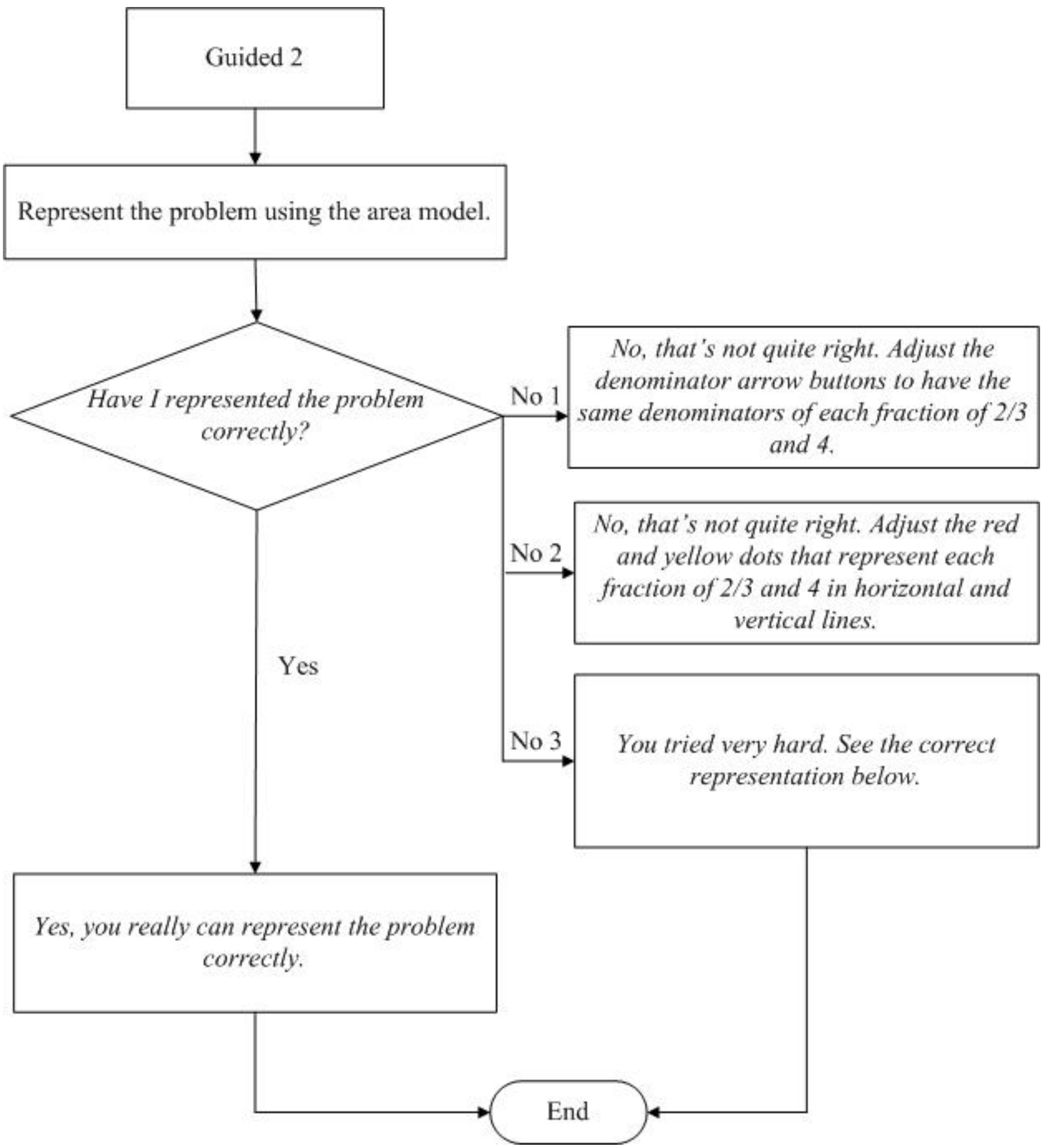

Start
Lessons
Lesson 1
Lesson 2
Lesson 3
Lesson 4
Lesson 5
Lesson 6
Lesson 7
Review
Modeling
Guided 1
Guided 2
End
End

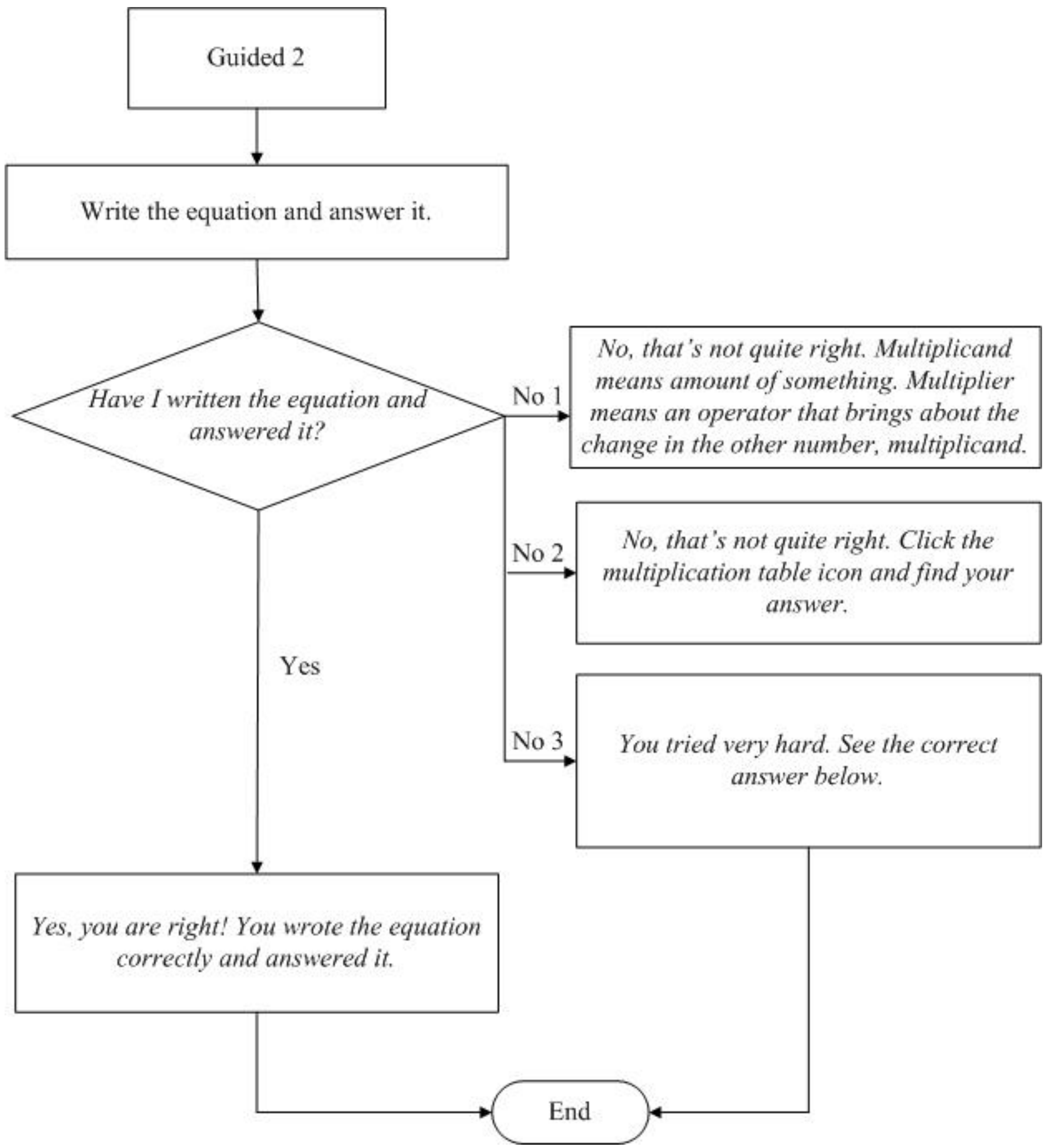

Guided 1
Read
I will read the problem. I will reread the problem if I don't understand it.
Read the problem.
Have I understood the problem and can now move forward?
No 1
Click the sound button and reread the problem.
No 2
Let's click the vocabulary button that you don't understand.
No 3
You tried very hard. Let's go to the next step (Restate)
Yes
Yes, you really understand the problem. Now, let's go to the next step (Restate).
Restate


Guided 1
Restate
I will find all important information in the problem and click and highlight it.
Click and highlight all importatn information.
Have I found all important information in the problem and clicked and highlighted it?
No 1
No, that's not quite right. Find the numbers and what you are asked to
No 2
No, that's not quite right. . Check the words that are repeated and find the quantities of them.
No 3
You tried very hard. See the important information.
Now let's go to the next step (Represent) and see why those numbers/words are important.
Yes
Yes, you really know important words.
Now, let's go to the next step (Represent)
Represent


Guided 1
Represent
I will represent the problem.
Represent the problem using the area model.
Have I represented the problem correctly?
No 1
No, that's not quite right. Adjust the denominator arrow buttons to have the same denominators of each fraction of 2/3 and 4.
No 2
No, that's not quite right. Adjust the red and yellow dots that represent each fraction of 2/3 and 4 in horizontal and vertical lines.
No 3
You tried very hard. See the correct representation below. Now let's go to the next step (Answer) and compute the equation.
Yes
Yes, you really can represent the problem correctly. Now, let's go to the next step (Answer).
Answer


Guided 1
Answer
I will write the equation and answer it.
Write the equation and answer it.
Have I written the equation and answered it?
No 1
No, that's not quite right. Multiplicand means amount of something. Multiplier means an operator that brings about the change in the other number, multiplicand.
No 2
No, that's not quite right. Click the multiplication table icon and find your answer.
No 3
You tried very hard. See the correct answer below. Now let's try guided 2 practice problems.
Yes
Yes, you are right! You wrote the equation correctly and answered it. Now let's try guided 2 practice problems.
End


Guided 2
Represent the problem using the area model.
Have I represented the problem correctly?
No 1
No, that's not quite right. Adjust the denominator arrow buttons to have the same denominators of each fraction of 2/3 and 4.
No 2
No, that's not quite right. Adjust the red and yellow dots that represent each fraction of 2/3 and 4 in horizontal and vertical lines.
No 3
You tried very hard. See the correct representation below.
Yes
Yes, you really can represent the problem correctly.
End



Guided 2
Write the equation and answer it.
Have I written the equation and answered it?
No 1
No, that's not quite right. Multiplicand means amount of something. Multiplier means an operator that brings about the change in the other number, multiplicand.
No 2
No, that's not quite right. Click the multiplication table icon and find your answer.
No 3
You tried very hard. See the correct answer below.
Yes
Yes, you are right! You wrote the equation correctly and answered it.
End

Appendix B: Students Accuracy Performances

Table G.1 *Tiffany's Accuracy Performance*

Lesson	Problem type	Question type	GP #	Scr	Phase															
					Baseline				Intervention											M
					1	2	3	A	4	5	6	7	8	9	10	11	12	13	A	14
1	Combine 1	Represent	1		I			0				C						I	33	
			2.1	C									I							I
			2.2																	
			2.3																	
		Equation	1					NR											60	
			2.1						I											
			2.2							C						C				
			2.3								C						I			
2	Combine 2	Represent	1		I			0				I							33	
			2.1			I							C							
			2.2											I						
			2.3																	
		Equation	1					0										C	50	
			2.1																	C
			2.2	I						I										
			2.3								C						I			
3	Partition 1	Represent	1		C			33											50	
			2.1	I		I														
			2.2				I							C						
			2.3												I					
		Equation	1					0				C						C	50	
			2.1										I							C
			2.2	I																
			2.3								I									
4	Partition 2	Represent	1					0								I			25	
			2.1	I																
			2.2				I							I						
			2.3						C						I					
		Equation	1	I	I			0				I							0	
			2.1			I							I							C
			2.2																	
			2.3																	

Table G.1 *Tiffany's Accuracy Performance (continued)*

Lesson	Problem type	Question type	GP #	Scr	Phase															
					Baseline				Intervention											M
					1	2	3	A	4	5	6	7	8	9	10	11	12	13	A	14
5	Compare 1	Represent	1					0		C						I			50	
			2.1	I													C			
			2.2				I													
			2.3						I											
		Equation	1		I			0											0	
			2.1			I							I							
			2.2	I										I						
			2.3												I					
6	Compare 2	Represent	1					NR			I						C		60	
			2.1															I		
			2.2																	
			2.3							C						C				
		Equation	1			C		50											33	
			2.1				I							I						
			2.2						C						I					
			2.3	C																
7	Compare 3	Represent	1					NR				C						C	50	
			2.1																	C
			2.2	I						I										
			2.3								I									
		Equation	1				I	0											50	
			2.1						I						I					
			2.2													C				
			2.3														C			
Number of correct				2	1	1	0		2	3	2	3	1	1	0	3	3	3		4
Accuracy percentage				20	20	20	0		40	60	40	60	20	20	0	60	60	60		80

Note. A = accuracy percentage; C = correct; GP = Guided Practice; I = incorrect; M = maintenance test; NR = not reported; Scr = screening test.

Table G.2 *John's Accuracy Performance*

Lesson	Problem type	Question type	GP #	Scr	Phase															
					Baseline					Intervention										M
					1	2	3	4	A	5	6	7	8	9	10	11	12	13	A	14
1	Combine 1	Represent	1		I				0			I						C	33	
			2.1	I									I							C
			2.2																	
			2.3																	
		Equation	1						100										75	
			2.1					C												
			2.2							I						C				
			2.3								C						C			
2	Combine 2	Represent	1		I				0			C							67	
			2.1			I							I							
			2.2											C						
			2.3																	
		Equation	1						NR									C	75	
			2.1																	C
			2.2	I						I										
			2.3								C						C			
3	Partition 1	Represent	1		C				33										100	
			2.1	I		I														
			2.2				I							C						
			2.3												C					
		Equation	1						NR			C						C	75	
			2.1										C							I
			2.2	I																
			2.3								I									
4	Partition 2	Represent	1						0							C			100	
			2.1	I																
			2.2				I							C						
			2.3					I							C					
		Equation	1	I	C				100			I							0	
			2.1			C							I							C
			2.2																	
			2.3																	

Table G.2 *John's Accuracy Performance (continued)*

Lesson	Problem type	Question type	GP #	Scr	Phase															
					Baseline					Intervention										M
					1	2	3	4	A	5	6	7	8	9	10	11	12	13	A	14
5	Compare 1	Represent	1						0	I						C			67	
			2.1	C													C			
			2.2				I													
			2.3					I												
		Equation	1		I				0										67	
			2.1			I							C							
			2.2	C										I						
			2.3												C					
6	Compare 2	Represent	1						NR		I						C		40	
			2.1															C		
			2.2																	
			2.3							I						I				
		Equation	1			C			33										50	
			2.1				I							I						
			2.2					I							C					
			2.3	I																
7	Compare 3	Represent	1						NR			I						C	25	
			2.1																	C
			2.2	I						I										
			2.3								I									
		Equation	1				I		50										100	
			2.1					C							C					
			2.2													C				
			2.3														C			
Number of correct				2	2	2	0	2		0	2	2	2	3	5	4	5	5		4
Accuracy percentage				20	40	40	0	40		0	40	40	40	60	100	80	100	100		80

Note. A = accuracy percentage; C = correct; GP = Guided Practice; I = incorrect; M = maintenance test; NR = not reported; Scr = screening test.

Table G.3 Alec's Accuracy Performance

Lesson	Problem type	Question type	GP #	Scr	Phase														
					Baseline				Intervention										M
					1	2	3	A	4	5	6	7	8	9	10	11	12	A	13
1	Combine 1	Represent	1		I			0				I						50	C
			2.1	C									C						
			2.2																
			2.3																
		Equation	1					NR										100	
			2.1						C										
			2.2							C						C			
			2.3								C						C		
2	Combine 2	Represent	1		I			0				I						67	
			2.1			I							C						
			2.2											C					
			2.3																
		Equation	1					NR										67	C
			2.1																
			2.2	I						I									
			2.3								C						C		
3	Partition 1	Represent	1		I			0										50	
			2.1	C		I													
			2.2				I							I					
			2.3												C				
		Equation	1					NR				C						67	C
			2.1										C						
			2.2	I															
			2.3								I								
4	Partition 2	Represent	1					0								C		75	
			2.1	I															
			2.2				I							I					
			2.3						C						C				
		Equation	1	I	I			0				C						100	
			2.1			I							C						
			2.2																
			2.3																

Table G.3 Alec's Accuracy Performance (continued)

Lesson	Problem type	Question type	GP #	Scr	Phase															
					Baseline				Intervention											M
					1	2	3	A	4	5	6	7	8	9	10	11	12	A		13
5	Compare 1	Represent	1					0		C						I		75		
			2.1	I													C			
			2.2				I													
			2.3						C											
		Equation	1		I			50										100		
			2.1			C							C							
			2.2	I										C						
			2.3												C					
6	Compare 2	Represent	1					NR			C						0	75		
			2.1																	I
			2.2																	
			2.3							C						C				
		Equation	1			C		50										100		
			2.1				I							C						
			2.2						C						C					
			2.3	I																
7	Compare 3	Represent	1					NR				C						100		C
			2.1																	
			2.2	C						C										
			2.3								C									
		Equation	1				I	0										100		
			2.1						C						C					
			2.2													C				
			2.3														C			
Number of correct				3	0	2	0		5	4	4	3	5	3	5	4	4			4
Accuracy percentage				30	0	40	0		100	80	80	60	100	60	100	80	80			80

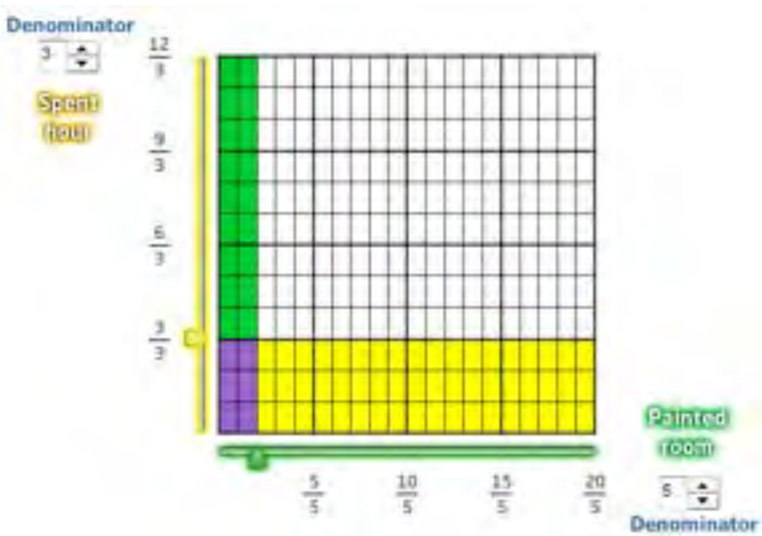
Note. A = accuracy percentage; C = correct; GP = Guided Practice; I = incorrect; M = maintenance test; NR = not reported; Scr = screening test.

Appendix C: Sample of Instructional Probes

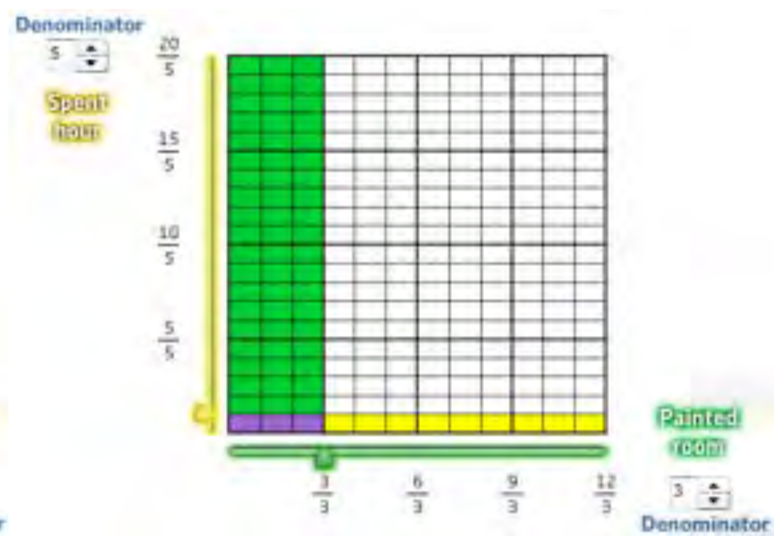
Which of the following area models best represents the problem? (1-3)

1 $\frac{1}{5}$ of a room can be painted in 1 hour. If 3 hours were spent to paint the room, how much of the room was painted?

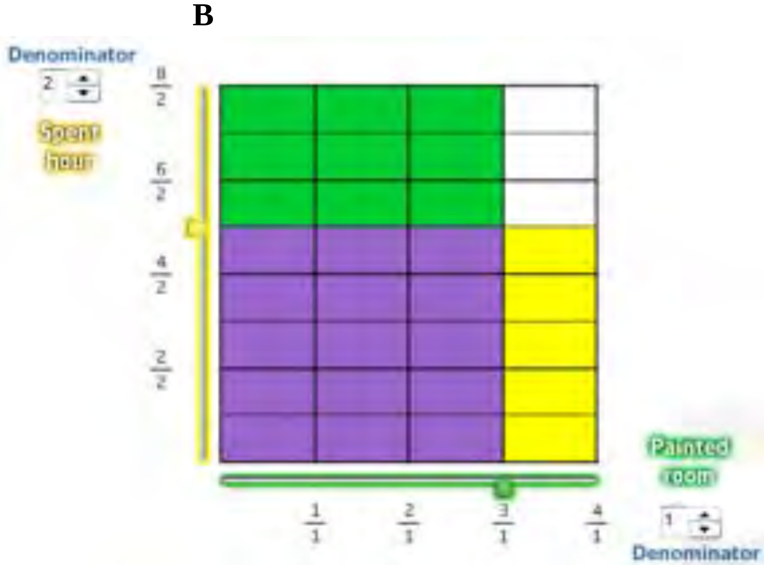
A



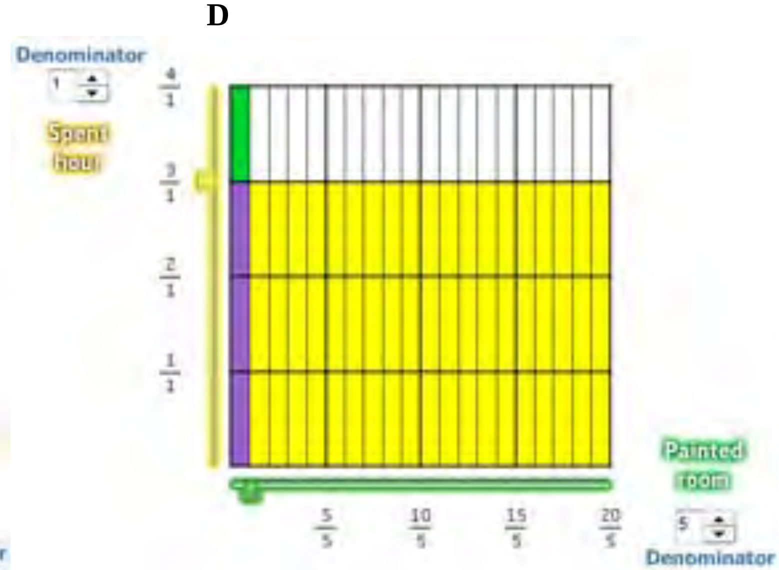
C



B

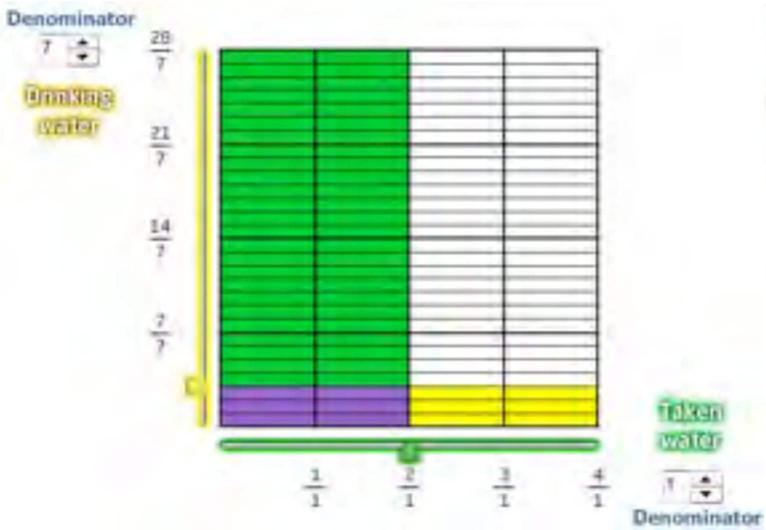


D

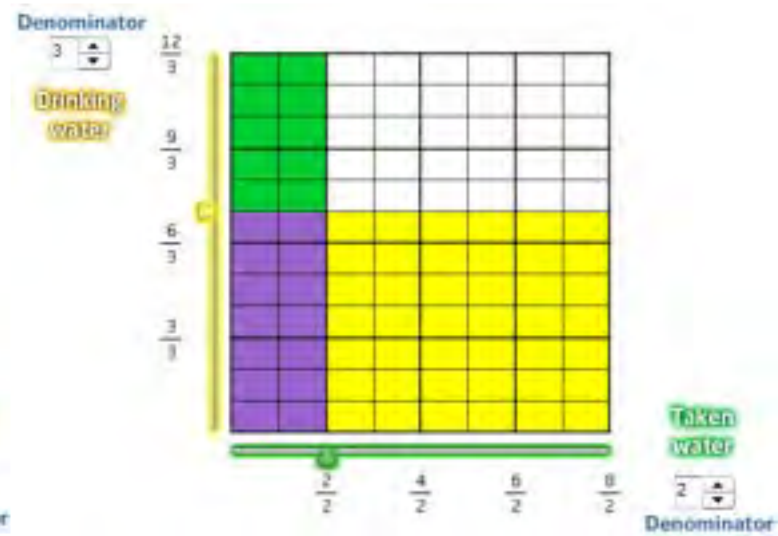


2 Min takes 2 pints of water for a hike up a mountain trail and back. She plans to drink $\frac{3}{7}$ of the water on the way up. How many pints does Min plan to drink on the way up?

A



C



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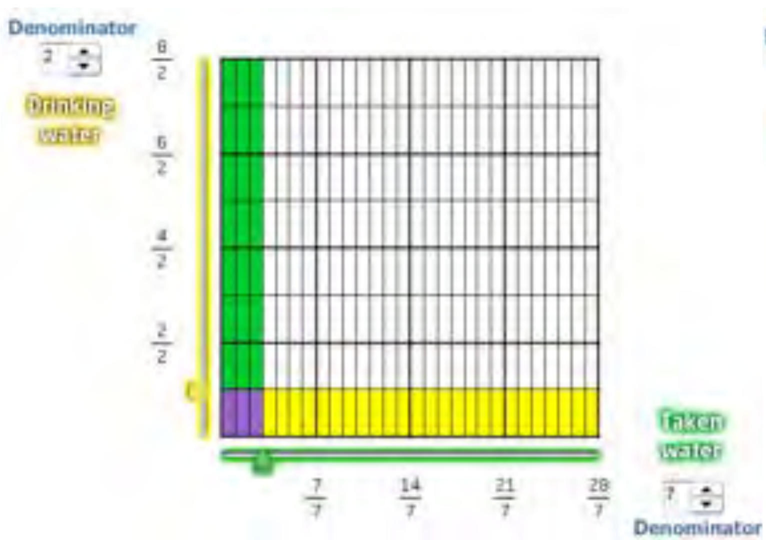
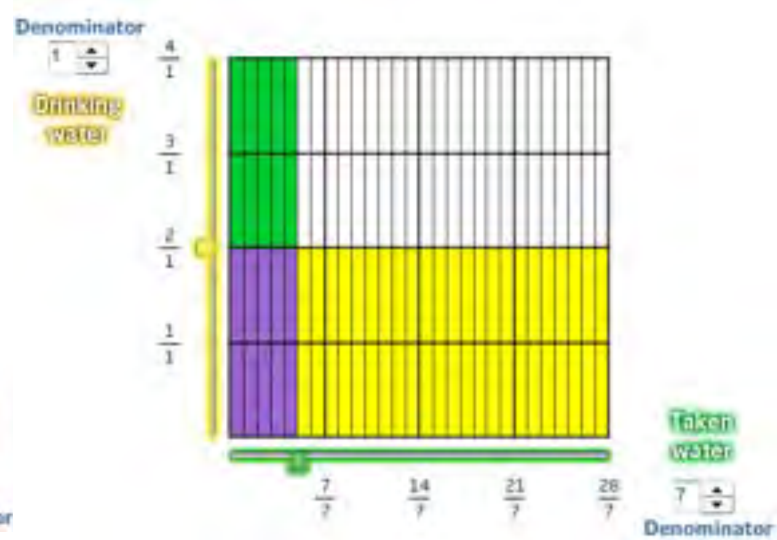
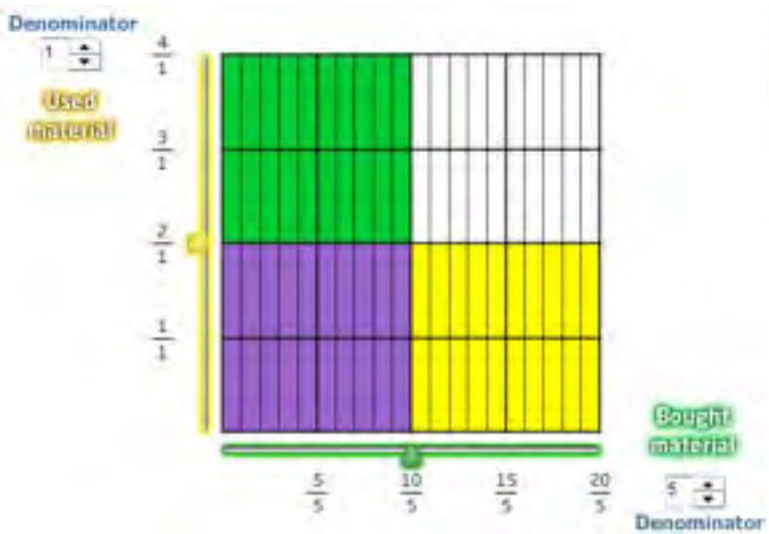


D

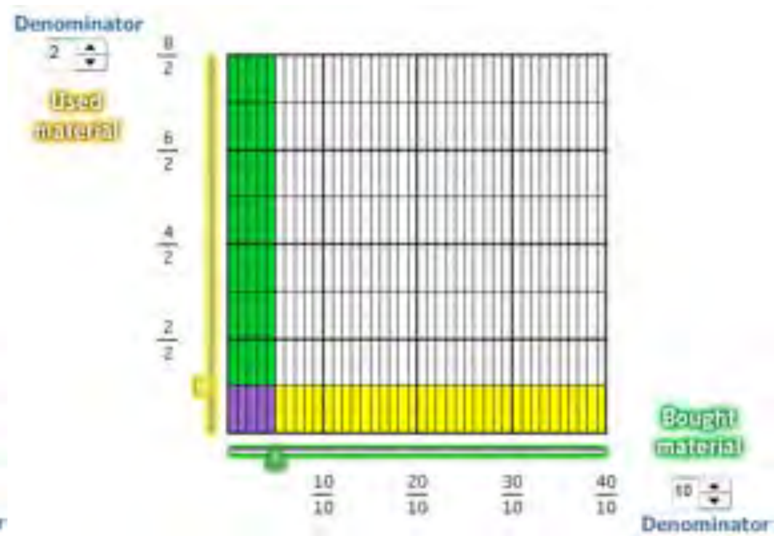


3 Colin bought $\frac{5}{10}$ of a yard of material for his class project. He only used $\frac{1}{2}$ of the material. How much material did Colin use for his project?

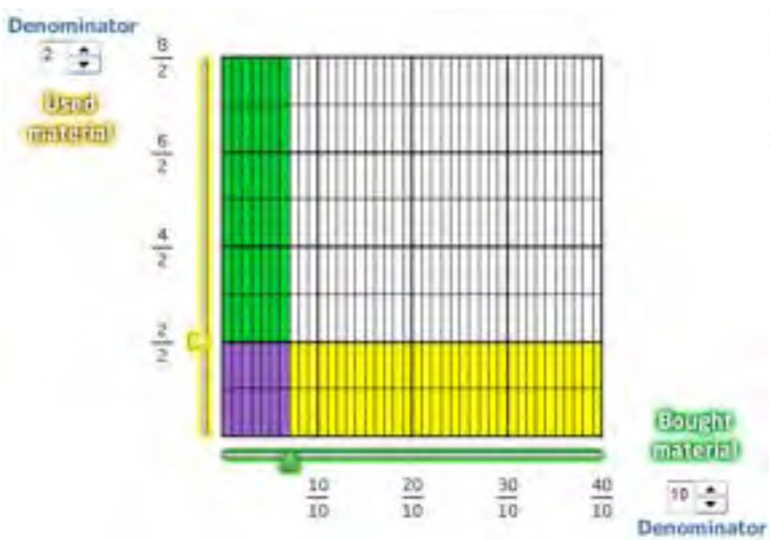
A



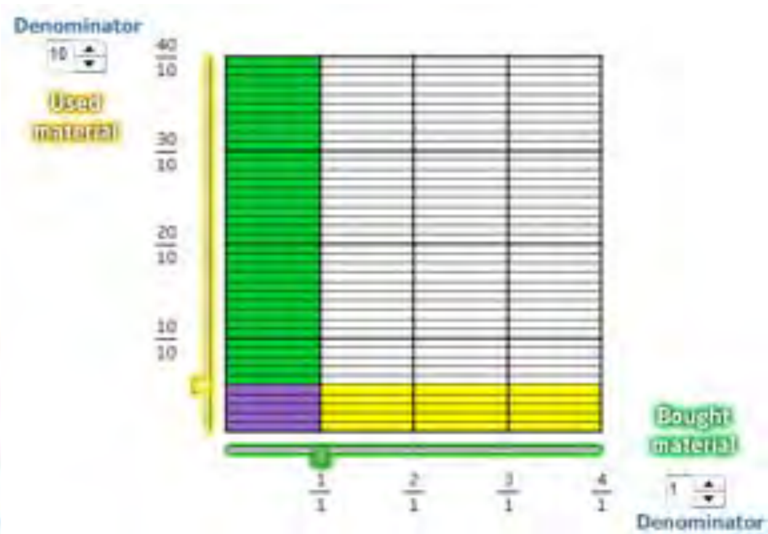
C



B



D



Which of the following equations best represents the problem? (4 – 5)

4 Of the students in the band, $\frac{3}{10}$ play the cello. Also, there are 3 times as many students who play the violin as students playing the cello. What is the fraction of the students playing the violin in the band?

A $\frac{3}{10} - \frac{1}{3} = \frac{2}{7}$

B $\frac{3}{1} \times \frac{3}{10} = \frac{9}{10}$

C $\frac{3}{10} \times \frac{3}{3} = \frac{9}{30}$

D $\frac{1}{3} \div \frac{5}{10} = \frac{10}{15}$

5 A chocolate cake recipe calls for 2 cups of chocolate powder. A chocolate cookie recipe calls for $\frac{2}{7}$ times as much chocolate powder as chocolate cakes. How many cups of chocolate powder are needed for making chocolate cookies?

A $\frac{2}{7} \times \frac{2}{1} = \frac{4}{7}$

B $\frac{1}{2} + \frac{3}{7} = \frac{13}{14}$

C $\frac{2}{7} + \frac{2}{2} = \frac{4}{9}$

D $\frac{2}{2} \times \frac{3}{7} = \frac{6}{14}$

Appendix D: Fidelity Checklists

A. Observation Information

Directions: Please complete the following information about the group you are observing.

1. Observer: __________________ 2. Lesson: ___________ 3. Student: ______________

4. Starting time: _________________ 5. Ending time: ____________________________

B. Fidelity Checklist Directions: Place a check beside the corresponding box to indicate if the student follows instructional procedures of the implementation.

For Lesson 1 through Lesson 7, the student	**Yes**	**No**	***Notes***
• Does 2-minute Multiplication Fact practices.			
• Goes to Modeling and watch the video.			
• Goes through all the procedures presented in Read strategy of Guided 1 practice.			
• Goes through all the procedures presented in Restate strategy of Guided 1 practice.			
• Goes through all the procedures presented in Represent strategy of Guided 1 practice.			
• Goes through all the procedures presented in Answer strategy of Guided 1 practice.			
• Goes through all the procedures presented in Question 1 of Guided 2 practice.			
• Goes through all the procedures presented in Question 2 of Guided 2 practice.			
• Goes through all the procedures presented in Question 3 of Guided 2 practice.			

For Review, the student	**Yes**	**No**	***Notes***
• Does 2-minute Multiplication Fact practices.			
• Goes through all the procedures presented in Question 1 of Review.			
• Goes through all the procedures presented in Question 2 of Review.			
• Goes through all the procedures presented in Question 3 of Review.			

Overall, how would you rate this student's fidelity for the intervention lesson? Circle one.

Excellent	**Good**	**Fair**	**Poor**
4	3	2	1

Appendix E: Student Social Vadility and Usability Questionnaire

A. **Instructions:** Think about *Fun Fraction* you used for solving word problems with fractions and multiplication. Please indicate your response to each item by circling one of the five responses to the right.

	Questions	Responses				
1.	Overall, Multiplication Fact helped me practice multiplication facts.	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
		5	4	3	2	1
2.	Overall, Vocabulary helped me review the definitions, representations, examples, and nonexamples of words for learning word problem-solving with fractions and multiplication.	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
		5	4	3	2	1
3.	Overall, Lessons helped me to better understand word problem-solving with fractions and multiplication.	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
		5	4	3	2	1
4.	Overall, Review helped me to review the lessons on word problem-solving with fractions and the multiplication.	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
		5	4	3	2	1

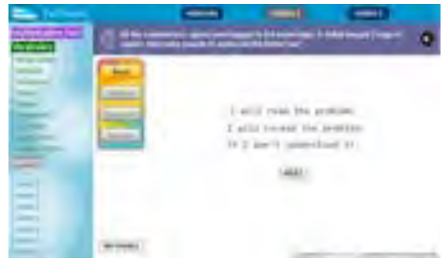
5.Overall, do you feel that you will use *Fun Fraction* in the future? Why or why not?

B. ***Fun Fraction* Evaluation Directions:** Be familiar with *Fun Fraction* for the evaluation. Please indicate your response to each item by circling one of the five responses to the right.

Information	**Responses**				
1. Are the number of practice problems appropriate for learning word problem-solving with fractions and multiplication?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	5	4	3	2	1
2. Can you easily identify tasks, activities, and contents on the website?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	5	4	3	2	1
3. Is the sequence of instruction on the website appropriate?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	5	4	3	2	1
Interface	**Responses**				
4. Is the design of the website consistent in terms of colors, font types, and font sizes?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	5	4	3	2	1
5. Does each page or each window have links that are easy to navigate?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	5	4	3	2	1
6. Does the design maintain attention for important information by using appropriate colors?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	5	4	3	2	1

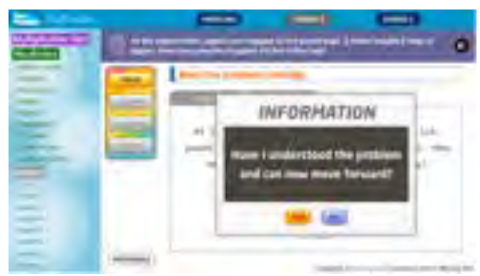
Interactivity	Responses				
7. Is there feedback to help you solve mathematics problems or tasks?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	5	4	3	2	1
8. Can you change area models by moving or clicking your mouse?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	5	4	3	2	1
9. Can you easily choose lessons that you want to learn?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	5	4	3	2	1

Appendix F: Perspectives on Cognitive and Metacognitive Strategies



1. What did you like best about learning and using *Read, Restate, Represent, Answer* strategies in *Fun Fraction*?

2. What was hard for you to learn and use *Read, Restate, Represent, Answer* strategies in *Fun Fraction*?



3.What did you like best about the *Information* message (e.g., asking you if you have understood the program and can now move forward) in *Fun Fraction* program?

4. What was hard for you regarding the *Information* message (e.g., asking you if you have understood the program and can now move forward) in *Fun Fraction* program?

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